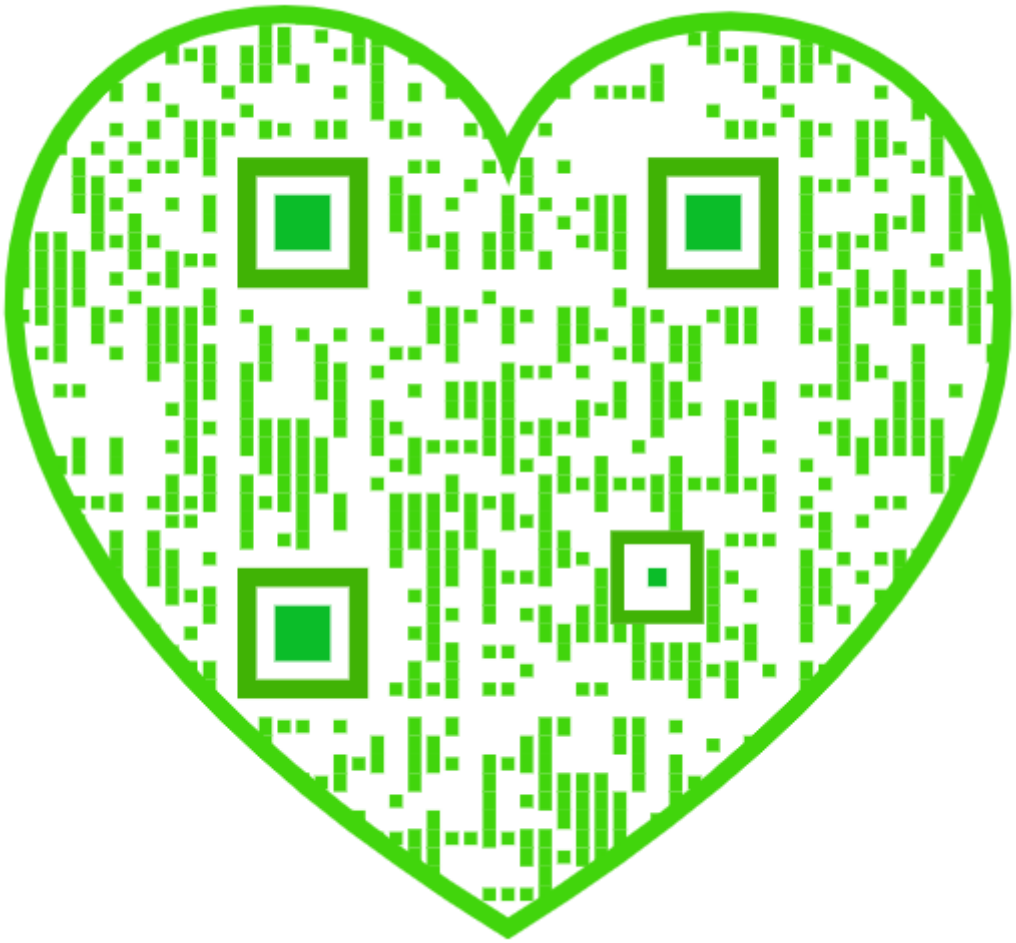

THE APPLETON SCHOOL BOOKS

ARITHMETIC

BY

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PREFACE

1. **The dominating thought** in the preparation of this series has been that not only is it one of the chief functions of the teaching of arithmetic to lead the child to think, but also that this thinking should usually relate to concrete materials adapted to his comprehension and interest. The controlling method is therefore inductive rather than deductive; concrete rather than abstract.

2. **The framework** around which the subject is built consists of the essential processes of arithmetic, which the child must learn to perform with accuracy, speed and intelligence. The authors have followed what they believe to be the most natural path to the accomplishment of these ends, *viz.*, that leading *from the concrete to the abstract.*

3. To be **really concrete** it is not sufficient to use named units instead of unnamed ones. If 379 + 486 is abstract, 379 bu. + 486 bu. is no less so. Data for problems have therefore often been sought in the child's own activities—weighing, measuring, observing, drawing, building, inquiring, reading.

4. While **motor activity** is emphasized, the manual work is not pushed to an extreme. No special material is required. Optional work is occasionally suggested, making use of simple materials and instruments easily procured or improvised.

5. **Exercises for abstract drill** have been included at suitable points. The teacher can readily dictate similar exercises if more are needed to secure the requisite practice.

6. **Related problems.** Usually the problems of each page are grouped about a *central thought*, so as to secure a certain unity in the subject-matter. The reviews are, of course, miscellaneous.

7. **Form study** thoroughly graded is made an integral part of the work.

8. **The commercial applications** receive due attention without being given exaggerated importance. Reduced *photographic fac-similes* of a number of business forms are given.

9. **Much information** is incidentally included in the problems.

10. **The problems are natural.** Factitious and unreal problems have been excluded. (E. g., "How many seconds in 8,372 mo.? How many oz. Troy in a long ton avoirdupois? Divide $\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{9}{16}$ of $\frac{14}{25}$ by $\frac{5}{8}$ of $\frac{3}{14}$ of $\frac{8}{15}$ of $\frac{33}{35}$.") The child's environment offers a wealth of material for problems relative to real conditions, more than ample for the development and mastery of all needed processes of arithmetic, and for the attainment of all the disciplinary benefits of its study.

11. **Obsolete** processes or types of problems have been omitted. The numerical needs of modern life are so many and so varied that no time can be spared for the antiquated or the artificial.

12. **Small numbers** are generally used. Every operation of arithmetic, however complex, consists of a succession of operations with very small numbers. If these are first thoroughly mastered, their repetition in longer problems offers no difficulty. Large numbers are not prematurely forced on the child, but, when prepared for them, he is led to them gradually and naturally in problems relating to actual conditions.

13. **Much oral work** precedes the written work in each topic.

14. **Frequent reviews** and occasional **general reviews** are given.

15. The **order of topics** has been determined to meet the needs of the child-mind. Its craving for variety and change is healthy and normal and must be heeded, but it can not be properly satisfied by a disjointed and fragmentary treatment. Nor, on the other hand, may the child-mind be forced to follow the practice of highly trained adult-minds in the complete and final treatment of each topic at one time. In due course, however, each leading subject is made the center of instruction, and its principles are summarized and emphasized.

16. The need to **teach through the eye** has been constantly borne in mind. The illustrations are, however, never simply pictorial, but always *illustrative* and usually integral parts of the text.

17. **Omissions** ranging from single problems or pages to entire chapters or topics can readily be made. Pages entitled "problems" present no new principles, and any such pages may be omitted without breaking the continuity of the course. A shorter course retaining all the characteristic topics may be formed by omitting some or all of pages 25, 27, 46–50, 66–71, 90, 91, 112, 116, 124–131, 142–144, 184, 185, 187–189, 191, 193–198, 201–205, 211, 214–218, 231–245.

THE AUTHORS.

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BOOK III

I

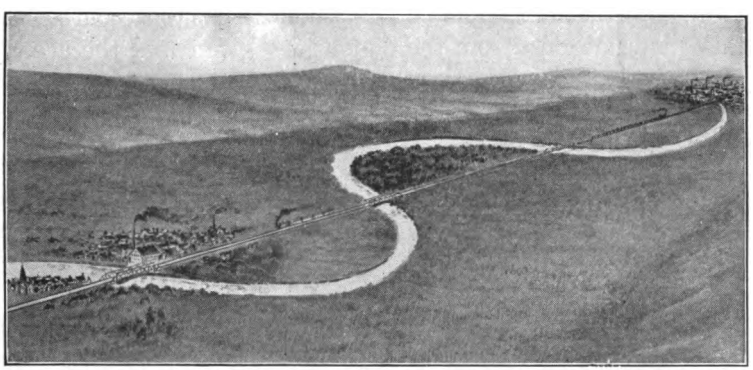
REVIEW AND PREPARATION

PROBLEMS—GEOGRAPHY

Oral.

1. How many bends does the river make between the two cities?

2. Taking each bend as a semicircle of radius 1 mile,



find the length of the river between the two cities. (Circumference = 3.14 times the diameter.)

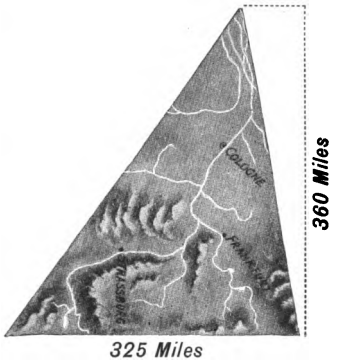
3. How many hours does it take a steamboat to go from one city to the other at the rate of 9.42 mi. an hour?

4. How many miles farther is it by boat than by rail between the cities?

5. This river, only a part of which is shown, drains an area equivalent to a rectangle 58 mi. long and 8 mi. wide; how many square miles of territory does it drain?

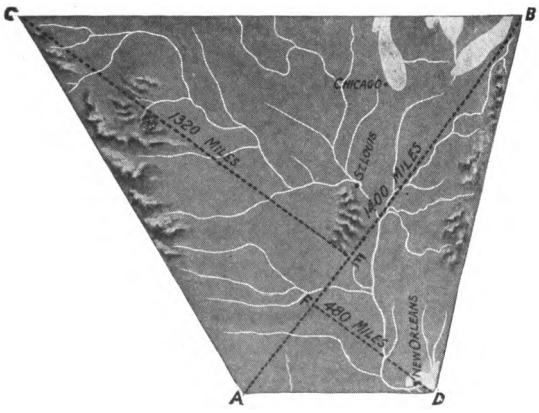
Written.

1. The approximate area drained by the river Rhine and its tributaries is represented by this triangle; from the dimensions given find this area in square miles.



2. The area of the state of New York is 52,170 sq. mi.; does the Rhine drain an area greater or less than the area of that state? By how many square miles?

3. The approximate area drained by the Mississippi and its tributaries is represented by this quadrilateral; from the dimensions given find the area.



4. The area of the United States to the nearest thousand is 3,026,000 sq. mi.; what part of this area does the Mississippi drain?

5. The area drained by the river Danube and its tributaries may be represented by two trapezoids, one having bases 200 mi. and 500 mi. and altitude 300 mi., the other having bases 200 mi. and 500 mi. and altitude 600 mi. Find the area of this territory.

6. If convenient, bring data and compute other geographical areas.

Written.

1. A family canned 40 cans of fruit at an average cost of $17\frac{3}{4}¢$ a can. The same fruit would have cost 25¢ a can at the store; how much was saved by canning it at home?

2. From a sack of flour costing $1.20 a family made 100 loaves of bread. The other ingredients (yeast, salt, milk, etc.) cost 75¢, and the fuel for baking 60¢. If loaves of the same size and quality cost 5¢ each at the bakery, how much did the family save on 100 loaves?

3. A laborer earns $500 a year and pays out 15% of it for rent; what is the rent each month?

4. A mechanic earns $900 a year and pays out 20% of it for rent; what monthly rental does he pay?

The table shows suggested divisions of incomes:

Yearly Income.	Food.	Clothing.	Rent.	Higher Life.	Savings.	Miscellaneous Expenses.
$1,000 to $2,000	25%	15%	20%	15%	—%	10%
800 to 1,000	30	15	—	15	10	10
500 to 800	—	10	15	10	10	10
500 and under	50	10	15	10	8	—

NOTE.—$500 to $800 is meant to exclude $500 but include $800. Similarly in the other cases.

5. Find the numbers to fill the blanks of the table.

6. According to the table, how many dollars would a family with an income of $2,000 pay for food in 1 year? For clothing? For miscellaneous expenses, that is, insurance, medicine, amusements, etc.? For higher life, that is, for charity, books, etc.?

7. Answer the same questions for an income of $800.

8. What monthly rental would a family with an income of $1,200 pay? What would be their weekly food bill?

9. Make and solve 5 other problems from the table.

Oral.

1. What part of 10 is 5? What is a short way to multiply a whole number by 5?

2. Multiply by 5 without writing the figures:

36; 18; 28; 49; 70; 136; 43; 476.

3. What part of 100 is 25? What is a short way to multiply a whole number by 25?

4. Multiply by 25 without writing the figures:

16; 17; 24; 40; 13; 19; 116.

5. 75 is what part of 100? If two ciphers are annexed to a whole number and $\frac{3}{4}$ of the result is taken, by what is the number multiplied?

6. Multiply by 75 without writing the figures:

12; 16; 40; 24; 20; 28; 56.

7. 125 is what part of 1,000? How may a whole number be multiplied easily by 125? Illustrate the method. How may a whole number be multiplied easily by 1.25?

Find the cost of:

8. 16 lb. of coffee at 25¢ a pound.

9. 36 cans of baking powder at 25¢ a can.

10. 84 cans of corn at 5¢ each.

11. 124 bottles of olives at 25¢ a bottle.

12. 82 doz. oranges at 50¢ a dozen.

13. 24 packages of rolled oats at 25¢ a package.

14. 136 Arithmetics at 75¢ each.

15. 48 chairs at $1.25 each.

16. 96 caps at 75¢ each.

17. 112 doz. teaspoons at $1.25 a dozen.

18. 320 pairs of skates at $1.25 a pair.

19. 444 pairs of slippers at 75¢ a pair.

Milk is shipped in cans from the farms to dairy stations in large cities, where it is bottled and distributed to consumers.

A teacher took her class to a dairy station and found that:

(1) Milk was received from the dairymen in 10-gallon cans.

(2) It was drawn from the railway station in loads of 24 cans each.

(3) It was put into pint or quart bottles by a bottling machine at the rate of 4 bottles a minute.

(4) The bottles were placed in crates containing 12 bottles each.

(5) Some delivery wagons carried 16 crates.

(6) The dairymen who supplied the milk received 4¢ a quart.

(7) It cost owners of the dairy station 2¢ a quart to handle the milk.

(8) The milk was retailed at 8¢ a quart.

Oral.

1. How many quarts of milk were there in each can shipped by the dairymen?

2. How many pint bottles would the milk from 1 can fill?

3. How many quarts are there in 10 crates of quart bottles? In 10 crates of pint bottles?

4. For how much does the milk from 1 crate of quart bottles retail?

5. What is the retail value of the milk from a 10-gallon can?

6. A family uses 2 quarts a day; the amount used in 30 days is equivalent to how many crates of quart bottles? Of pint bottles?

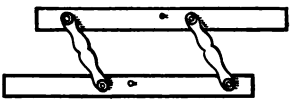
7. A boarding-house uses a crate of quart bottles daily; how many gallons does it use in a week?

Written.

8. What is the retail value of the milk in a wagon carrying 16 crates of quart bottles? Of one carrying 12 crates of quart bottles and 8 crates of pint bottles?

9. Make and solve other problems, using the data above.

Drawing Parallels.—Parallel lines may be drawn by means of two rulers fastened together as shown. Such an instrument is called *parallel rulers.* The crosspieces are of equal length and the four joints are movable. Parallel rulers may be made easily out of cardboard.



Oral.

1. Name two figures that have their opposite sides parallel.

2. State how to draw a parallelogram with parallel rulers.

3. Do lines *a* and *b* of Figure 1 appear parallel?

Fig. 1.

4. By use of parallel rulers or by measuring the distance between the lines at different places determine whether or not the lines are parallel.

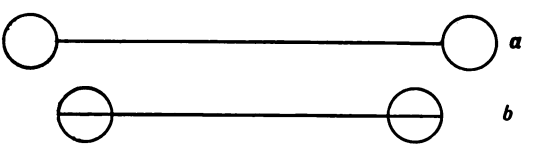


Fig. 2.

5. Which is the longer, line *a* or line *b* in Figure 2? In Figure 3? Test by measuring them.

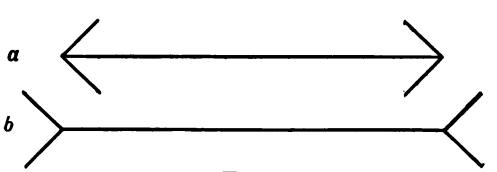


Fig. 3.

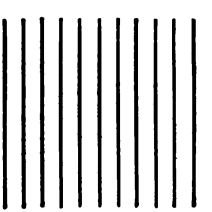
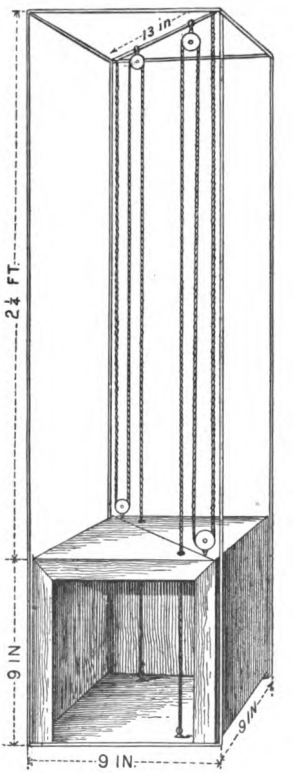


Fig. 4.

6. Make an exact copy of Figure 4 and turn it so that the lines are in a horizontal position. Which figure looks the wider? The higher? How do they compare by measurement?

Written.

1. Each of 15 pupils in manual training made a weaving loom. The base frame of each loom required two pieces of moulding $1\frac{1}{2}$ ft. long and two pieces $\frac{3}{4}$ ft. long; how many feet were left out of six 12-foot strips?



2. Harold made a model of an elevator. The framework was made of stiff wire. Find from the given dimensions how many feet of wire were used. Find how many feet of string he used for the cables.

3. He noticed that he worked on the model $1\frac{3}{4}$ hr. on each of 3 days, $1\frac{1}{2}$ hr. on each of 2 days, and $\frac{3}{4}$ hr. on each of 5 days; how long did it take him to make it? How many days of 8 hr. each?

4. A class in domestic science made 16 cup-custards. For each cup they allowed 1 egg, 1 oz. of sugar, and $\frac{1}{2}$ pt. of milk. At 18¢ a dozen for eggs, 6¢ a pound for sugar, and 6¢ a quart for milk, what did all of these materials cost?

5. A class of girls made 5 sachet bags, 8 calendars, and 7 letter cases for an exhibition. Each sachet bag required $1\frac{1}{2}$ ft. of ribbon, each calendar $\frac{3}{4}$ ft., and each letter case $1\frac{2}{3}$ ft.; how many feet of ribbon did they use?

6. The boys made bent iron candlesticks, each requiring $1\frac{3}{8}$ yd. of iron. They used $12\frac{3}{8}$ yd.; how many candlesticks did they make?

1. What may be used to draw a circle?

2. What is a diameter of a circle? A radius?

The diameters of round bodies, such as pencils, ink bottles, and water glasses, may be measured by use of triangles and a scale as shown in Figure 1.

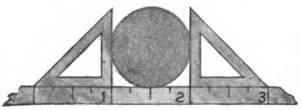


Fig. 1.

3. Measure the diameter of several such objects and record the results.

Fig. 2.

4. Wrap a piece of paper about each object until the ends overlap, as in Figure 2. Prick the ends with a pin. Release the paper and measure between the pinholes, as in Figure 3. This is the distance around the object.

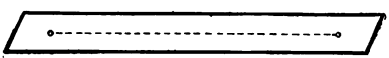


Fig. 3.

5. Find the ratio of the length around each object to its diameter. Record these results.

6. By how much does each result differ from 3.14?

It has been stated in Book II that the ratio of the length of a circle (circumference) to its diameter can be found approximately only, and that it is represented by π. Use 3.14 as the value of π in the following problems.

7. The diameter of a circle is 2 in.; by what must this be multiplied to find the circumference? Find this length.

Written.

8. Find the circumference of a circle of diameter 6 in.; of diameter 1 ft.; of radius 6 in.; of radius 3 yd.

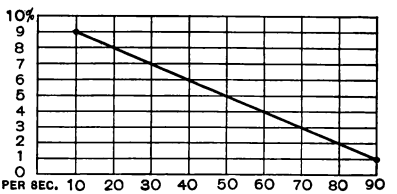
9. If the circumference of a circle is given, by what must it be divided to produce the diameter?

10. Find the diameter of a circle of length 3.14 ft.; find the length of a radius.

11. Find the length of the radius of a circle whose circumference is 6.28 in.; of one whose circumference is 9.42 yd.

When water flows through a ditch some of it soaks into the soil. The quantity depends upon the character of the soil, the rate of flow and other conditions. For soil of a certain character the diagram shows what per cent of the water entering the ditch soaks into the soil during the passage through the ditch at different rates of flow.

For example, when 10 cu. ft. of water enter during a second, 9% of it soaks into the soil.



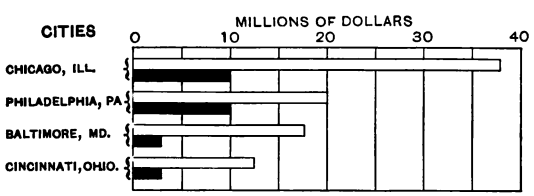
Oral.

1. What per cent is lost when 20 cu. ft. enter per second? 40 cu. ft.? 80 cu. ft.? 100 cu. ft?

2. When 10 cu. ft. enter per second, how many cubic feet soak into the soil? How many cubic feet are delivered at the other end of the ditch per second? Per minute?

3. Make and solve 5 similar problems.

4. In the diagram the white strips represent the number of million dollars' worth of men's clothing recently manufactured during one year in the cities named; the black strips represent similarly the value of women's clothing. What was the value of Chicago's product of each kind? Of Philadelphia's? Of Baltimore's? Of Cincinnati's?



Written.

5. Represent similarly:

	MEN'S CLOTHING.	WOMEN'S CLOTHING.
New York City.....	$110,000,000	$105,000,000
Boston.....	15,000,000	5,000,000
St. Louis.....	10,000,000	5,000,000

II

FUNDAMENTAL PROCESSES

NOTATION AND NUMERATION

Notation and Numeration. The method of writing numbers is called *notation* and that of reading them *numeration.*

Periods. For convenience in reading, numbers are set off into *periods* of three digits each, counting both ways from the decimal point. The last period may have fewer than three digits.

A comma (*separatrix*) or a larger space than that between the other figures may be used to separate two periods.

Names of Periods. The first three periods of integers have already been named units, thousands, millions. The fourth period is called *billions.*

Thus, $2,630,000,000 is read: *two billion six hundred thirty million dollars.*

Decimal Places. The decimal places in order to the right from the decimal point are named: *tenths, hundredths, thousandths, ten thousandths,* etc.

Decimal places beyond the fourth are rarely needed.

1. Read the numbers in the table showing the number of pounds of freight that passed through the various waterways in a recent year:

WATERWAYS.	POUNDS.	TONS.
Suez	8,560,284,000	——
Welland	869,596,000	——
Sault Ste. Marie	15,062,594,000	——
Detroit River	25,845,696,000	——

2. State the number of tons, or tonnage, for each waterway named in Exercise 1.

Table of Periods and Places. The following table shows the names of the periods and places from millionths to billions:

Places	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units	*Decimal point*	Tenths	Hundredths	Thousandths	Ten thousandths	Hundred-thousandths	Millionths
	5	0	7	2	3	7	6	8	9	5	2	1	·	0	2	6	7	4	5
Periods	Billions			Millions			Thousands			Units				Thousandths			Millionths		

Oral.

1. Read the number in this table. How many periods are there in the integral part? In the decimal part?

Read:

2. Some light waves are .00011 in. long; others are .00001 in. long.

3. Sea water is 1.0249 times as heavy as fresh water.

4. The length of a circle is nearly 3.14159 times the length of its diameter.

5. Light waves whose lengths are greater than 0.000029 in. or less than .000013 in. are not visible to the eye.

6. In a recent year some of the imports and exports of the United States were:

IMPORTS.		EXPORTS.	
Sugar	3,031,915,875 lb.	Cotton	3,500,778,763 lb.
Coffee	1,091,004,252 lb.	Wheat	154,856,102 bu.

7. The total exchanges made by the clearing houses of the United States were:

1899	$88,909,661,776	1901	$114,616,277,512
1900	84,556,685,444	1902	116,021,618,003

8. Read similar facts from newspapers and magazines.

Written.

1. When numbers are written for adding, how should the corresponding orders stand?

2. Add 200, 1,760, 25.7, and 2.005.

3. How may the work of addition be tested? Test Exc. 2.

4. The number of people working in the lumber business in the United States in a certain year was:

In furniture factories	58,719	In saw mills	100,087
" cooper shops	37,226	" planing mills	61,600
" piano factories	6,220	" other factories	82,390

What was the total number?

5. Find the total amount of receipts of a school system from the following school funds:

Instruction fund	$440,976.12
Building fund	120,000.00
Repair fund	25,046.75
Library fund	5,138.70
Contingent fund	144,411.50

6. Recently the governmental departments of the State of New York had on hand the following sums:

Department of public instruction	$81,000.00
Department of public works	69,374.62
Department of charities and correction	24,600.00
Department of public safety	21,817.92
Legislative department	12,000.00
Department of law	8,000.00
Comptroller's office (estimated)	5,000.00
Treasurer's office	3,000.00
Department of assessment and taxation	395.00
Fire marshal	400.00
Civil service commission	300.00
Park commission	300.00
Mayor's office	300.00
Municipal court	25.00
Police court	25.00

Find the total amount on hand.

Oral.

1. Add the digits in the number 2,639. Divide the sum by 9. What is the remainder or excess?

Casting out Nines. Dividing the sum of the digits of a number by 9 to find the remainder is called *casting out the nines.*

Cast out the nines from these numbers and find the excesses:

2.	9,910.	3.	3,061.	4.	72,320.
5.	56,432.	6.	3,136.	7.	125,963.
8.	172,360.	9.	40,372.	10.	14,037.
11.	125,678.	12.	1,265,384.	13.	69,327,436.

The work of addition may be tested by casting out nines.

The sum of the excesses of the addends must equal the excess of the result, or if the sum of these excesses is greater than 8, its excess must equal that of the result.

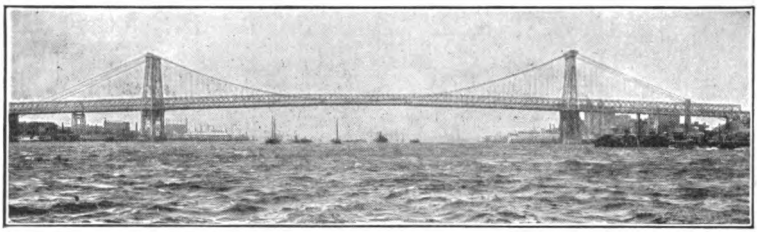
The process is the same when the addends contain a decimal part.

	Excess.
26731	1
37654	7
64385	8
2635	7
3094	7
4265	8
9994	4

Written.

Add and test the results by casting out nines:

14.	63450	15.	76327	16.	26932
	85709		94308		26872
	36245		86347		26462
17.	260032	18.	180060	19.	163.006
	310032		19760		29.376
	187625		4330		197.263
	243394		308699		90.008
20.	347798	21.	198.221	22.	225.449
	543297		768.993		539.117
	936720		231.795		200.791
	399444		550.117		661.331



THE WILLIAMSBURGH BRIDGE. Length, 7,264 feet.

Written.

1. The following are the approximate amounts of the contracts for constructing the new East River, or Williamsburgh, bridge from Manhattan to Brooklyn:

For the	Manhattan tower foundation....	$373,462.70
" "	Brooklyn " "	485,082.70
" "	Manhattan anchorage.....	797,770.00
" "	Brooklyn "	771,778.00
" "	Manhattan approach.....	1,464,000.00
" "	Brooklyn "	947,000.00
" "	suspended structure.....	1,123,400.00
" "	steel towers and end spaces.....	1,221,726.00
" "	minor work	1,500,000.00

Find the total cost of the bridge according to the data given. Test the accuracy of your work by casting out nines.

2. The total length of the Brooklyn bridge is 5,989 ft.; how much longer is the Williamsburgh bridge?

3. State a practical method of testing subtraction. Apply the process in testing your result for Exercise 2.

4. The value of the clay pottery made in a recent year was:

Ohio.....	$10,519,138	West Virginia	$ 1,166,464
New Jersey.....	6,192,959	All other States....	(?)
Pennsylvania.....	1,876,263	United States.....	24,127,453

Answer the question asked in the table and test your work by casting out nines.

Written.

1. 9 = 10 minus what? What must be taken from 10 times a number to leave 9 times the number? State a short way to multiply a number by 9.

2. Multiply by 9: 23; 54; 76; 97.

3. Find the cost of 47 packages of macaroni at 9¢ a package.

4. Find the cost of 381 packages of breakfast food at 9¢ a package.

5. 99 = 100 minus what? What must be taken from 100 times a number to leave 99 times the number? State a short way to multiply by 99.

6. Multiply by 99: 14; 27; 59; 84.

7. Find the cost of 52 shirt waists at 99¢ each.

8. Find the cost of 16 pairs of curtains at 99¢ a pair.

9. State a short way to multiply a number by 999.

10. Multiply by 999: 4; 51; 84; 26; 718.

11. State a short way to multiply by 98.

12. Multiply by 98: 7; 15; 36; 49; 72.

13. A man sold 12 horses at $98 each; how much did he receive for the horses?

14. State a short way to multiply by 199.

15. Multiply by 199: 3; 9; 52; 69; 81; 46.

16. Find the cost of 42 barrels of flour at $4.99 a barrel.

17. State a short way to multiply by 198.

18. Multiply by 198: 6; 12; 93; 47.

19. Find the cost of 7 rugs at $1.98 each.

20. Multiply by 298: 3; 16; 39; 57.

21. Find the cost of 16 doz. handkerchiefs at $2.98 a dozen.

22. A milliner sold 24 hats at $3.98 each; how much did she receive for the hats?

(1) When several factors are to be combined, the amount of work sometimes depends upon the order in which they are taken.

Form the products as indicated:

(1) $(5 \times 7) \times 6 = —$. (2) $5 \times (7 \times 6) = —$. (3) $(5 \times 6) \times 7 = —$.

Which is the easiest to do without writing?

Oral.

Multiply:

1. $2 \times 9 \times 5$.	2. $7 \times 4 \times 5$.	3. $14 \times 4 \times 5$.
4. $11 \times 18 \times 5$.	5. $5 \times 28 \times 3$.	6. $82 \times 2 \times 5$.
7. $2 \times 17 \times 3 \times 5$.	8. $8 \times 13 \times 5$.	9. $25 \times 84 \times 8$.

(2) For rapid oral work, it is often easier to multiply successively by the factors of the multiplier.

Form the products as indicated:

(1) $14 \times 15 = —$. (2) $7 \times (2 \times 15) = —$.

Which is the easier to do without writing?

Form the products as indicated:

(1) $34 \times 35 = —$. (2) $2 \times 17 \times 5 \times 7 = (2 \times 5) \times 17 \times 7 = —$.

Which is the easier to do without writing, 34×35 or $(2 \times 5) \times 17 \times 7$?

Multiply:

10. 16×45.	11. 18×35.	12. 12×45.
13. 24×50.	14. 36×50.	15. 22×45.
16. 38×50.	17. 150×18.	18. 35×16.
19. 25×14.	20. 162×15.	21. 25×84.

22. A teamster hauled 24 cu. ft. of earth at a load; how many cubic feet did he haul in 45 loads?

23. There are 12 pairs of shoes in a case; how many pairs are there in 25 cases?

24. Which is the easier to do, 15×48 or $5 \times 48 \times 3$?

Oral.

1. What is the area of a square whose side is 4 in.?

2. What is the square, or second power of 3? Of 5?

3. What is the volume of a cube whose edge is 4 in.?

4. What is the cube, or third power, of 2? Of 3? Of 5?

The product of two like factors, as 2×2, 3×3, 4×4, 5×5, is written for brevity 2^2, 3^2, 4^2, 5^2. Similarly, $2 \times 2 \times 2$, $3 \times 3 \times 3$, $4 \times 4 \times 4$, and so on, may be written 2^3, 3^3, 4^3, and so on. Likewise $a \times a$, or aa, is written a^2, and $bcc = bc^2$.

Exponents. The small figure written above is called an *exponent* and shows how often the number above which it stands is taken as factor in the product.

5. What exponent is used to indicate the square of a number? Indicate by exponents the square of 6, 7, 8, 10.

6. State the product of the factors in 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2.

7. If s is the side of a square, what product represents its area? Denote this product by use of an exponent.

8. What exponent is used to indicate the cube of a number? Indicate thus the cube of 5; 6; 7; 8; 10; 15.

9. If e is the edge of a cube, what product represents its volume? Indicate this product by use of an exponent.

10. State the product of the factors in 1^3, 2^3, 3^3, 4^3, 5^3.

Written.

11. What is the value of: $2^3 \times 3^2$? 5×4^3? $3^2 \times 10^3$?

12. Write the factors of the following numbers, using exponents where possible: 25, 50, 75, 18, 27, 63, 72, 90.

13. $600 = 6 \times 10 \times 10$. Write this by use of an exponent.

14. Write by use of exponents: 300; 900; 1,000; 5,000.

15. Write with exponents $a \times a$, $b \times b$, $c \times c$, aaa, aab, abb, rr, arr.

16. If $a = 2$, $b = 4$, $c = 3$, what is the value of a^2b? a^2c^3? ab^2? $3a^3b^2$?

Fundamental Property of Division. In every division the relation holds:

Divisor times quotient equals dividend.

Thus, $47 \div 5 = 9\frac{2}{5}$, and $5 \times 9\frac{2}{5} = 47$.

Find the numbers to fill the blanks:

	1.	2.	3.	4.	5.	6.
Dividend:	27	46	62	—	—	$\frac{2}{5}$
Divisor:	4	—	12	$2\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$
Quotient:	—	$5\frac{3}{4}$	—	7	$\frac{5}{6}$	—

If fractions are not admitted to the quotient a remainder may occur. In this case the relation is:

Divisor times quotient plus remainder equals dividend.

Thus, in $47 \div 5$, the quotient is 9 and the remainder 2, and $47 = 9 \times 5 + 2$.

Find the numbers to fill blanks below:

	7.	8.	9.	10.	11.	12.	13.
Dividend:	46	52	—	—	96	128	256
Divisor:	8	—	6	7	—	—	16
Quotient:	—	4	18	12	12	25	—
Remainder:	—	0	0	4	0	3	—

14. In these problems compare the dividends; the divisors; the quotients: $.4\overline{).8}$ $4\overline{)8}$

15. How does multiplying both dividend and divisor by 10 affect the quotient? How does multiplying them by 2 affect the quotient? Illustrate the latter case.

Multiplying or dividing both dividend and divisor by the same number does not change the quotient.

In the following problems compare the dividends; compare the divisors; compare the quotients:

16. $680 \div 40$; $68 \div 4$.

17. $384 \div 12$; $38.4 \div 1.2$.

18. $7200 \div 600$; $72 \div 6$.

19. $136 \div 8$; $34 \div 2$.

Oral.

Divide by 20:	1. 63.4.	2. 1.75.	3. 5.6.
	4. 18.4.	5. 40.3.	6. .104.
Divide by 30:	7. 2.7.	8. 1.96.	9. 38.52.
	10. .69.	11. 30.3.	12. 27.06.
Divide by 15:	13. 4.5.	14. 1.05.	15. 7.5.
	16. 46.5.	17. 900.	18. 5.25.
Divide by 200:	19. 64.8.	20. 126.	21. 7.26.
	22. 19.1.	23. 420.	24. .42.
Divide by 600:	25. 2.46.	26. 720.	27. 82.2.
	28. 954.	29. 660.	30. 17.4.

Written.

The work in division, as in dividing 34.65 by 7.7, may be arranged thus:

$$\begin{array}{r} 4.5 \\ 77\overline{)346.5} \\ 308 \\ \hline 38.5 \\ 38.5 \\ \hline \end{array}$$

What change is made in the dividend and divisor before dividing?

Perform the following divisions similarly:

31. $17.55 \div .39$. 32. $190.9 \div .83$. 33. $1.909 \div 2.3$.

34. $3.465 \div 6.3$. 35. $284.9 \div 3.7$. 36. $64.48 \div .31$.

Find the price of each to the nearest cent if:

37. 36 sheep cost \$107.
38. 24 handkerchiefs cost \$1.92.
39. 48 chairs cost \$60.
40. 6 barrels of flour cost \$33.
41. 160 baskets cost \$157.
42. 16 cans of corn cost \$1.24.
43. 74 books cost \$27.
44. 46 pairs of gloves cost \$51.
45. 50 hats cost \$93.
46. 28 ostrich plumes cost \$94.
47. 20 watches cost \$195.
48. 5 mandolins cost \$46.25.

49. In the preceding problems how is the position of the decimal point of the quotient determined?

Oral.

1. What are even numbers? Odd numbers? In what digits do even numbers end? Odd numbers?

2. Is 37 divisible by 3? Is the sum of its digits divisible by 3? In the same way test 49; 57; 63; 76; 81; 89; 96.

Test for Divisibility by 3. An integer *is divisible by 3 if the sum of its digits is divisible by 3*, otherwise not.

3. Is 316 divisible by 4? Is the number represented by its last two digits [16] divisible by 4? Test similarly 1,524.

Test for Divisibility by 4. An integer *is divisible by 4* if the *number represented by its last two digits is divisible by 4*, otherwise not.

Which of the following are divisible by 3? By 4?

4. 128. 5. 276. 6. 648. 7. 87,000.

8. 585. 9. 3,284. 10. 950. 11. 11,656.

Which of the following could be set in 3 equal rows? In 4?

12. 126 cabbage plants. 13. 1,706 tomato plants.

14. 872 fruit trees. 15. 1,000 pansies.

16. Name some multiples of 5. Is 45 divisible by 5? 50? 73? 90? 99? 109? 160?

Table of Tests for Divisibility. The preceding and other tests for divisibility are collected in the following table.

A number is divisible:

By 2, if it ends in 0, 2, 4, 6, 8.

By 3, if the sum of its digits is divisible by 3.

By 4, if the number represented by the two digits at the right is divisible by 4.

By 5, if it ends in 0 or 5.

By 6, if divisible by 2 and by 3.

By 8, if the number represented by the three digits at the right is divisible by 8.

By 9, if the sum of its digits is divisible by 9.

By 10, if it ends in 0.

By 12, if it is divisible by 3 and by 4.

For 7 and 11, actual division is the best test.

Oral.

1. By use of the tests of the preceding page, name the numbers of the following list divisible by 2; by 3; by 4; etc. 16, 21, 48, 35, 52, 17, 64, 128, 400, 536, 279, 1436.

2. Find among the following a number divisible only by itself and 1: 4, 12, 17, 25, 38, 51.

Prime and Composite Numbers. A number which is divisible only by itself and 1 is called a *prime number*. If divisible by any other number it is called a *composite number*.

Every number can be expressed as a product of prime factors.

The prime factors may be found by dividing the number by any prime factor recognized by the above methods, then dividing the quotient similarly by any of its prime factors, and so on, until a prime quotient is reached. The original number is the product of all the divisors and the last quotient. Thus, $60 = 2 \times 2 \times 3 \times 5$.

3. Express all the numbers to 100 as products of prime factors. Make a list of the prime numbers less than 100.

Common Multiples. A number that is divisible by two or more numbers is called a *common multiple* of those numbers. The least one is called the **least common multiple** (l. c. m.).

Thus, 12 is a common multiple of 2 and 3. 12, 24, and 36 are common multiples of 4, 6, and 12; and 12 is the least common multiple.

4. Find the *least* common multiple of: 8, 12; 3, 6; 6, 8; 5, 4; 3, 5.

When not readily seen by inspection, the l. c. m. may be found by factoring the given numbers. The l. c. m. will be the product obtained by taking each prime factor as often as it occurs in any of the given numbers. Thus, since $28 = 2 \times 2 \times 7$ and $42 = 2 \times 3 \times 7$, the l. c. m. of 28 and 42 is $2 \times 2 \times 3 \times 7 = 84$. The l. c. m. contains 2 twice, because it is found in 28 twice. It contains 3 and 7 each once, because they occur once only in one or both of the given numbers.

Written.

Find the l. c. m. of:

5. 24, 56. 6. 18, 27. 7. 14, 35. 8. 32, 48.

Written.

1. In a recent year the pay-roll of the city of New York contained the names of 43,000 persons. The total amount of the pay-roll was about $38,000,000. Find within one cent the average compensation of each employee.

```
   883.72
43)38000.000
   344
   360
   344
   160
   129
   310
   301
    90
    86
```

Fixing the Decimal Point. The decimal point of the quotient will stand over the decimal point of the dividend, if the first figure of the quotient is placed over the last figure of the first partial dividend.

2. Show that this plan is followed in the work of Exercise 1.

3. How may the result be tested?

Find the average cost per mile of each ship canal; test your work:

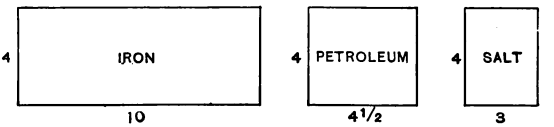
CANAL.	LENGTH.	COST.
4. Suez	90 miles	$100,000,000
5. Russian	16	10,000,000
6. Manchester	35.5	75,000,000
7. Kaiser Wilhelm	61	40,000,000
8. Elbe and Trave	41	6,000,000
9. Welland	27	30,000,000
10. Panama (proposed)	46	184,000,000

11. The table shows approximately the population and assessed value of property in New York city from 1870 to 1900:

YEAR.	POPULATION.	ASSESSED VALUE.
1870	1,500,000	1,300,000,000
1880	1,900,000	1,400,000,000
1890	2,500,000	2,100,000,000
1900	3,400,000	3,600,000,000

Find to the nearest cent the value of the property for each person (per capita).

1. These rectangles represent for a recent year the world's production of the minerals named.



Compute the areas of these figures. Each number found is the number of million tons of the mineral named. What amount of each mineral was produced?

2. In a recent year the acreage of corn, wheat, and oats and the number of bushels grown in the United States were:

	ACRES.	BUSHELS.	BUSHELS PER ACRE.
Corn.	83,328,000	2,105,103,000	———
Wheat.....	42,504,000	———————	12.3
Oats.....	———	809,280,000	30.0

Find the numbers to fill the blanks in the table.

3. If 1 sq. in. represents 1,000 acres, how long must a rectangle 2 ft. wide be to represent the acreage of corn? Of wheat? Of oats?

4. Find the average yield of potatoes per acre from the following table for each group of states and for the whole country:

GROUP OF STATES.	ACRES.	TONS OF POTATOES.
New England.....	180,000	505,000
Middle	410,000	1,440,000
Southern.....	240,000	420,000
Central.....	1,710,000	2,710,000
Pacific.....	110,000	290,000

5. Using $\frac{1}{8}$ in. to represent 5,000 tons, draw on the blackboard parallel lines to represent these yields.

Oral.

1. What is meant by numeration? By notation?

2. Name the four periods into which integers of 12 places may be separated for reading.

3. Name the two periods into which decimals of 6 places may be separated for reading.

4. State and illustrate a way of testing the work of addition. State another way.

5. State and illustrate a way of testing the work of subtraction.

6. State and illustrate two short processes in the work of multiplication.

7. How may the number of decimal places in a product be determined? In a quotient?

Written.

8. Write by use of exponents: $3 \times 3 \times 2$; $3 \times 3 \times 2 \times 2$; $3 \times 3 \times 3$; $aaab$; $aaabb$; $bbba$.

9. Write as a product of factors, using exponents where applicable: 50; 200; 1,000; 5,000; 8,000.

10. 9,945 tons of iron ore were unloaded from the lake steamer Wolvin in $4\frac{1}{2}$ hours; how many tons were unloaded per hour?

11. The table shows the number and value of the railway cars manufactured in the United States in a recent year:

KIND OF CAR.	NUMBER.	TOTAL VALUE.
Passenger coach	181	$957,526
Sleeping car	194	2,767,061
Dining car	37	404,503
Freight car	116,590	62,161,013

Find the average value of a car of each kind (to the nearest $100).

III

FORM STUDY AND MEASUREMENT

CIRCLES

Instruments: Compasses and Ruler

1. With compasses draw a circle of radius 3 in.

2. Draw a straight line 4 in. long. With the ends of the line as centers draw two circles each of radius 2 in.

3. Draw a circle of radius 1 in. to touch a circle of radius 3 in. How far apart are their centers? (Two answers).

4. The guns of a fort have a range of 3.5 mi. in all directions. Using 1 inch to represent a mile, draw a diagram to show the area in which an enemy would be exposed to the fire of the guns.

5. Forts Hamilton (H) and Wadsworth (W) protecting New York harbor are 1 mile apart. Some guns of H have a range of $4\frac{1}{2}$ mi., those of W $4\frac{3}{4}$ mi. Using $\frac{1}{2}$ in. to represent 1 mi., make a diagram to show the area covered by the guns of each fort.

6. Shade or color differently the areas covered by the guns of W alone, by those of H alone, by those of both forts.

7. Find by measurement the greatest distance that a ship sailing in a straight line would be exposed to the fire of both forts.

8. A square lawn 100 feet on a side is watered by a hose which can throw water 60 ft. Draw different figures showing the part that can be watered from one corner; from the middle of the side; from the center of the square.

1. If the diameter of a circle is 4 inches, what is the circumference of the circle? (See page 8.)

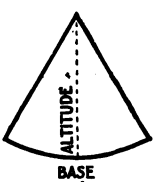
Fig. 1.

2. Cut out of paper a circle of radius 3 inches and cut along the lines marked in Figure 1. Open out the circle as shown in Figure 2. The parts are called *sectors*.

Fig. 2.

Sectors. A sector of a circle is the figure formed by two radii and an arc of the circle.

A small sector resembles a triangle, the arc being the base and a radius the altitude.



Area of Sector. It is shown in geometry that the area of a sector is found in the same way as that of a triangle.

Denoting the base of a sector by b and its altitude by r, the area is $\frac{br}{2}$.

Area of Circle. As all the sectors of a circle have the same altitude, the area of all of them is the sum of their bases times half of the common altitude. The sum of all the bases is the circumference, since the circumference was cut up to make them. Therefore, *Area of circle* $= 2\pi r \times \frac{r}{2} = \pi r^2$.

In words: **The area of a circle is π times the square of the radius.**

3. Find the length of a circle of radius 4 in., using 3.14 for π; find its area.

4. Find the area of circles of radii: 6 in.; 1 ft.; 2 in.; 2 ft.; 10 ft.; 10 in.

5. Wendell cut from cardboard a circle of radius 3 in. and a square 3 in. on a side. He weighed each and found that the square weighed 1 oz. and the circle slightly over 3 oz. Show that his results agree with the statement that the area of a circle is πr^2.

1. Find the area enclosed by a semicircle and a diameter, the diameter being 5 ft.

2. The rectangular court shown in the picture is broken by a circle in the center, by semicircles at the ends, and by a trapezoid at the side. From the dimensions given in feet find the area of the court not occupied by these spaces.

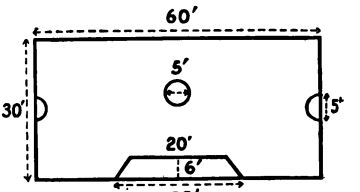


Fig. 1.

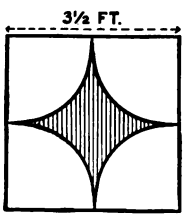


Fig. 2.

3. If a figure is drawn with dimensions as shown by Figure 2, what is the area of the square?

4. What part of a circle occupies each corner? What is the area of these parts?

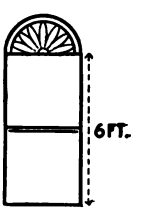


Fig. 3.

5. Draw a figure like that shown for Exercise 3, using a square of side 6 in. What is the area of the shaded part?

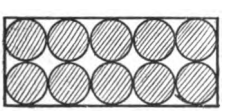


Fig. 4.

6. Find the area of the window, including the semicircular top.

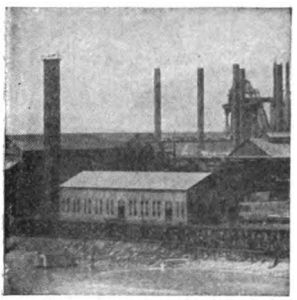
7. If the circular tiles shown in Figure 4 are 1 in. in diameter, what is the area of that part of the rectangle not covered by the tiles?

8. A walk surrounds a circular grass plat, as shown in the figure. Find the area of the plat; the area of the circle composed of the plat and the walk; the area of the walk.

Fig. 5.

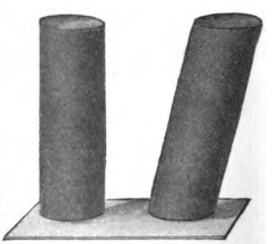
9. How many times will a bicycle wheel 28 in. in diameter revolve in going one mile?

Cylindrical Surfaces. Curved surfaces, like those of sections of stovepipe, factory smokestacks, and funnels of ships are called *cylindrical surfaces.*



Cylinders. If the ends of a cylindrical surface are in parallel planes the whole is called a *cylinder.* The ends are called the **bases.** If the base is a circle the cylinder is called **circular.**

Right and Oblique Cylinders. If the curved surface is perpendicular to the plane of the base, the cylinder is called a *right cylinder.* Cylinders which are not right cylinders are *oblique.*



Right. Oblique.

We shall treat only right circular cylinders, and shall call them cylinders simply.

1. Wrap a piece of paper around a cylinder. Cut it so that the edges meet. Trim the ends even with the cylinder. Unroll the paper. What kind of a figure is the paper?

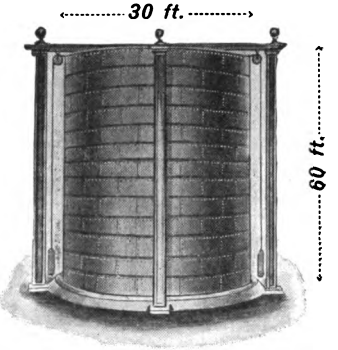
2. Measure the length and breadth of the paper and find the area. What is the area of the curved surface of the cylinder? How does the length of the paper compare with the distance around the cylinder?

Lateral Surface. The surface of an object exclusive of its ends is usually called its *lateral surface.*

Find the numbers to fill the blanks:

	DIAMETER.	ALTITUDE.	LATERAL AREA.
3.	4 in.	10 in.	——
4.	2 in.	——	31.4 sq. in.
5.	——	5 in.	62.8 sq. in.

1. Find the area of the curved surface of the cylindrical gas tank shown in picture.



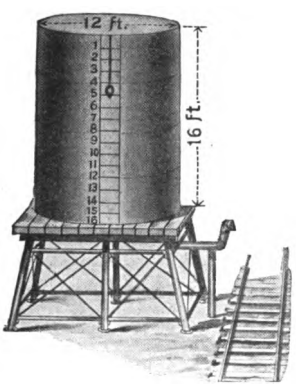
2. What will it cost to paint the surface mentioned in 1 at $.50 a square yard? Test.

3. How many square feet are there in the top and curved surface of a gas tank 36 ft. in diameter and 45 ft. high?

4. What will it cost to paint the top and curved surface of the tank of Exc. 3 at $.45 a sq. yd.?

The volume of a cylinder is the product of its altitude and the area of its base.

5. How many cu. ft. in a cylindrical tank 1 ft. high, whose base covers 40 sq. ft.? In a tank 2 ft. high having the same base? 5 ft. high?



6. Find the area of the base of the water tank shown in the picture. From the height given find the volume in cubic feet.

7. Taking 1 cu. ft. as $7\frac{1}{2}$ gal. how many gallons does this tank hold?

8. When the gauge reads 5 it means that the water is 5 ft. deep in the tank; how many gallons are there in it then?

Find the numbers to fill the blanks for these cylinders:

	DIAMETER.	ALTITUDE.	VOLUME.
9.	3 in.	8 in.	—
10.	—	5 in.	62.8 cu. in.
11.	1 ft.	—	12.56 cu. ft.

1. What is the area of a square 4 in. on a side?

2. If a circle is drawn in a square, as shown in Figure 1, what is the area of the circle?

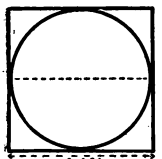


Fig. 1.

3. What is the area of the part of the square not included in the circle?

4. Figure 2 shows a walk about a semicircular garden. Find the length of the outside of the walk; of the inside.

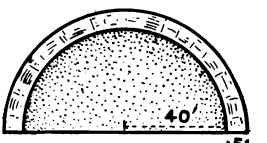


Fig. 2.

5. Find the area of the garden and the walk; of the garden alone; of the walk.

6. An assembly hall is in the form of a rectangle with one semicircular end. The rectangle is 20 ft. by 40 ft. What is the cost of boards for the floor at $18.20 a thousand feet?

The following table relates to circles; find the missing numbers (using $\pi = 3.14$):

	Diameter.	Radius.	Circumference.	Area.
7.	17 in.	——	——	——
8.	——	3.5 ft.	——	——
9.	——	——	——	3.14 sq. ft.
10.	——	——	18.84 ft.	——

The following table relates to cylinders; find the missing numbers:

	Diameter.	Altitude.	Lateral Area.	Total Area.	Volume.
11.	10 in.	12 in.	——	——	——
12.	6 in.	——	150.72 sq. in.	——	——
13.	——	8 ft.	——	——	226.08 cu. ft.
14.	——	10 in.	62.8 sq. in.	——	31.4 cu. in.

IV

FRACTIONS

REDUCTION

Oral.

1. How may $\frac{2}{4}$ be changed to $\frac{4}{8}$? $\frac{1}{3} = \frac{?}{6}$. $\frac{2}{3} = \frac{?}{6}$.

2. What effect is produced upon the value of a fraction by multiplying or dividing both terms by the same number?

3. How may a fraction be changed to an equal fraction with a different denominator?

4. $\frac{1}{2} = \frac{?}{8}$. 5. $\frac{1}{2} = \frac{?}{6}$. 6. $\frac{1}{2} = \frac{?}{10}$.

7. $\frac{1}{3} = \frac{?}{6}$. 8. $\frac{1}{3} = \frac{?}{9}$. 9. $\frac{1}{3} = \frac{?}{18}$.

10. 3 lb. and 4 oz. are how many ounces? In what unit are they expressed before adding?

11. How must numbers be expressed before they can be added or subtracted?

12. What does the word denominator mean? What does it name or designate?

Fractional Unit. Any one of the equal parts of a unit, or thing, is called a *fractional unit.*

Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{100}$ are fractional units.

13. What fractional unit is found in each of the following fractions? $\frac{2}{3}$; $\frac{5}{7}$; $\frac{9}{11}$; $\frac{13}{27}$; $\frac{99}{100}$; $\frac{17}{34}$; $\frac{29}{58}$; $\frac{43}{49}$.

14. Fractions expressed in the same fractional unit are alike in which term?

15. What change must be made in fractions with different denominators in order to make them fractions having the same fractional unit?

Fractions having the same fractional unit are added or subtracted by adding or subtracting their numerators and placing the result over the common denominator.

Oral.

1. When the denominators of several fractions are alike, what are they said to have?

Least Common Denominator. To change several fractions to others having a common denominator, it is necessary that the common denominator be a multiple of the given denominators; for simplicity, their l. c. m. is usually taken. This is called the *least common denominator* (l. c. d.) of the fractions.

Thus, the l. c. m. of the denominators of $\frac{1}{3}$, $\frac{2}{5}$, $\frac{7}{15}$ is 15, hence it is selected as the common denominator.

2. By what must the denominator of $\frac{1}{3}$ be multiplied to make 15? How is this number found? By what number must the numerator then be multiplied so that the fraction may still equal $\frac{1}{3}$?

To reduce fractions to fractions having the least common denominator. Find the l. c. m. of all the denominators, divide it by each denominator and multiply both terms of the corresponding fraction by the quotient.

Thus, in $\frac{2}{3}$, $\frac{11}{12}$, $\frac{7}{15}$, the least common denominator is 60.

$60 \div 3 = 20$ $\quad$ $60 \div 12 = 5$ $\quad$ $60 \div 15 = 4$

$\frac{20 \times 2}{20 \times 3} = \frac{40}{60}$ $\quad$ $\frac{5 \times 11}{5 \times 12} = \frac{55}{60}$ $\quad$ $\frac{4 \times 7}{4 \times 15} = \frac{28}{60}$

3. Change $\frac{1}{2}$, $\frac{1}{9}$ to fractions having the denominator 162. What is their least common denominator?

4. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$ to fractions having the common denominator 30. Why can not a number less than 30 be taken?

Written.

Find the l. c. d. of:

5. $\frac{3}{5}$, $\frac{6}{7}$, $\frac{1}{10}$.

6. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{6}$.

7. $\frac{1}{7}$, $\frac{2}{3}$, $\frac{8}{9}$.

8. $\frac{3}{5}$, $\frac{1}{15}$, $\frac{2}{3}$.

Change to fractions having the l. c. d. and add:

9. $\frac{2}{5}$, $\frac{7}{8}$, $\frac{3}{10}$.

10. $\frac{3}{5}$, $\frac{1}{9}$, $\frac{2}{7}$.

11. $\frac{3}{4}$, $\frac{3}{7}$, $\frac{4}{5}$.

12. $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{7}$.

Oral.

1. What are mixed numbers? How are they usually added?

2. Add $17\frac{3}{4}$ and $5\frac{3}{4}$. Reduce the fractional part of the result to the lowest terms. What is the simplest form of the result?

3. Subtract $5\frac{3}{4}$ from $17\frac{1}{4}$.

$$\begin{array}{r} 17\frac{1}{4} = 16\frac{5}{4} \\ 5\frac{3}{4} = 5\frac{3}{4} \\ \hline \text{Diff.} = 11\frac{2}{4} = 11\frac{1}{2}. \end{array}$$

If the fraction in the minuend is less than the fraction in the subtrahend, what change must be made? Subtract the integers and fractions separately. How are the results combined?

4. Add $17\frac{2}{5}$, $5\frac{1}{3}$, 10, $8\frac{1}{2}$.

$$\begin{array}{r} 17\frac{2}{5} = 17\frac{12}{30} \\ 5\frac{1}{3} = 5\frac{10}{30} \\ 10 = 10 \\ 8\frac{1}{2} = 8\frac{15}{30} \\ \hline \text{Sum} = 40\frac{37}{30} = 41\frac{7}{30} \end{array}$$

If the fractions are expressed in different fractional units, what change must be made in the fractions? How is the work completed?

5. To what kind of fraction may a mixed number be changed?

6. In what way, different from that of Exercises 3 and 4, may mixed numbers be added and subtracted?

In adding or subtracting mixed numbers the integers and fractions may be treated separately and the results combined, or the mixed numbers may first be reduced to improper fractions.

Written.

7. From a roll of cloth containing $17\frac{5}{8}$ yd., $9\frac{3}{5}$ yd. are cut; how many yards remain in the roll?

8. Two garments, one requiring $3\frac{4}{5}$ yd. and another $5\frac{2}{3}$ yd. are cut from a piece of cloth containing $10\frac{1}{2}$ yd.; how many yards are left?

9. $20\frac{8}{9} + 119\frac{7}{18} = -$.

10. $705\frac{11}{12} - 126\frac{3}{4} + \frac{2}{3} = -$.

11. $100\frac{9}{10} + 999\frac{17}{20} = -$.

12. $12\frac{6}{7} + 18\frac{3}{14} + \frac{9}{28} = -$.

13. $146\frac{2}{3} - 123\frac{5}{18} = -$.

14. $145\frac{2}{3} + 9\frac{1}{6} + 3\frac{4}{5} = -$.

15. $17\frac{1}{6} + \frac{5}{6} - 4\frac{5}{9} = -$.

16. $40\frac{3}{25} + 1\frac{4}{5} - 9\frac{7}{10} = -$.

The following are recent prices of grain per bushel on the New York market. The months indicate when the grain is to be delivered to the purchaser. The prices are those prevailing at the opening and at the closing of the market; also the highest and lowest on that day.

WHEAT:	Open.	High.	Low.	Close.
May	95	95	95	95
July	$91\frac{1}{2}$	$91\frac{11}{16}$	$91\frac{5}{16}$	$91\frac{5}{8}$
Sept.	$83\frac{7}{8}$	$84\frac{1}{8}$	$83\frac{3}{4}$	$83\frac{7}{8}$
Dec.	84	84	$83\frac{1}{2}$	$83\frac{5}{8}$
CORN:				
May	$59\frac{3}{4}$	$60\frac{5}{8}$	$59\frac{1}{4}$	60
July	$53\frac{3}{8}$	$53\frac{7}{8}$	$53\frac{1}{4}$	$53\frac{7}{8}$
Sept.	$52\frac{1}{4}$	$52\frac{5}{8}$	$52\frac{1}{4}$	$52\frac{5}{8}$

Oral.

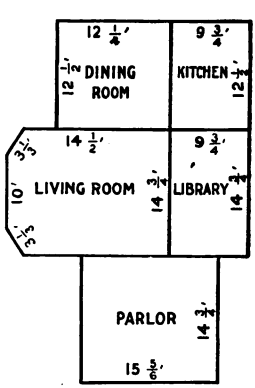
1. What was the difference between the opening and closing prices of July wheat? Between the highest and lowest prices?

2. Answer the same questions for September wheat.

3. How much was gained on 1,000 bushels of July wheat bought at the opening price and sold at the closing price?

4. Make and solve 4 other problems about grain. If convenient, use quotations like the above from a daily paper.

Written.



5. Find the number of feet of baseboard needed for each room in the plan, making no allowance for openings.

6. How many feet more are needed for the parlor than for the library?

7. Find the total length of the house, counting the thickness of each wall (outer or partition) as $\frac{1}{2}$ ft.

8. Make and solve 3 other problems about the plan.

Oral.

1. Find $\frac{2}{3}$ of : 9, 15, 17, $\frac{3}{5}$, $\frac{2}{3}$.

2. Compare 5×8 with 8×5. What effect has the order of the factors on the product? Similarly, compare $\frac{1}{2}$ of $\frac{2}{3}$ with $\frac{2}{3}$ of $\frac{1}{2}$.

Order of Factors.—The product is the same in whatever order the factors are taken.

3. Rose has $\frac{2}{3}$ of a yard of ribbon and used $\frac{7}{8}$ of it; what part of a yard did she use?

4. What is the meaning of $\frac{7}{8} \times \frac{2}{3}$? Of $\frac{3}{4} \times 2\frac{1}{2}$?

The product of any two fractions is a fraction whose numerator is the product of the given numerators and whose denominator is the product of the given denominators.

Mixed numbers must first be reduced to improper fractions.

5. How is the value of a fraction affected when both of its terms are divided by the same number? What is this process called?

Written.

Multiply (canceling when possible):

6. $24 \times \frac{3}{4}$.
7. $\frac{3}{4} \times 24$.
8. $\frac{2}{9} \times 18$.
9. $\frac{2}{3} \times 5$.
10. $\frac{5}{6}$ of $\frac{9}{25}$.
11. $\frac{2}{3}$ of $\frac{5}{8}$.
12. $\frac{5}{7} \times \frac{2}{3}$.
13. $\frac{3}{16} \times \frac{4}{9}$.
14. $\frac{5}{8} \times \frac{3}{10}$.
15. $\frac{3}{5} \times \frac{7}{8}$.
16. $\frac{9}{16} \times 4\frac{2}{3}$.
17. $16 \times 2\frac{2}{7}$.

18. How is the product of several fractions formed?

19. Find the product of $\frac{3}{4}$, $\frac{5}{9}$, $\frac{9}{10}$, and $\frac{4}{15}$. Cancel.

20. Mr. Morgan received $\frac{3}{5}$ of the income of a farm; he spent $\frac{5}{12}$ of his share. What part of the whole income from the farm did he spend?

21. When the engineer of a certain engine shuts off steam and puts on the brakes, the driving wheels of his engine still revolve $16\frac{1}{2}$ times before the engine comes to a standstill; if the driving wheels are $20\frac{2}{3}$ ft. in circumference, in what distance is the engine stopped?

Written.

1. The silver dollar weighs 412.5 grains. $\frac{9}{10}$ of its weight is pure silver; how many grains of pure silver are there in one dollar?

2. A half-eagle, or 5-dollar gold piece, weighs 129 grains. $\frac{9}{10}$ of its weight is pure gold; how many grains of gold are there in a half-eagle? How many grains of cheaper metal?

3. Gold is weighed by Troy weight, in which 5,760 grains = 1 pound. The gold in a half-eagle (see Exercise 2) is what part of a pound Troy?

4. The 5-cent piece weighs 77.16 grains. $\frac{1}{4}$ of each coin is nickel and the rest copper; how many grains of each kind are there in a 5-cent piece?

5. A builder bought a lot in New York City and after building his block found that the lot did not include a narrow strip along the street. This strip was $\frac{7}{8}$ in. wide and 40 ft. long; what was its area in square inches? He bought the strip for $50; how much was this per square inch? Per square foot?

6. At the rate found in Exercise 5, what would be the cost of a city lot 25 ft. by 100 ft.?

7. To allow for shrinkage, patterns for cast-iron articles are made $\frac{1}{8}$ in. longer per foot than the article. Similarly, for width and thickness. What are the length and width of a pattern for a cast-iron article $3\frac{1}{2}$ ft. long and 8 in. wide?

The allowance per foot for shrinkage in other castings is:

Brass	$\frac{3}{16}$ in.	Tin	$\frac{1}{12}$ in.
Lead	$\frac{1}{8}$ in.	Zinc	$\frac{3}{16}$ in.

8. A casting of lead is to be 18 in. long and 15 in. wide; what are the length and breadth of the pattern?

9. Make and solve 4 other problems about patterns for castings.

Oral.

1. What is the quotient of 6 books divided by 3 books?

2. What is the quotient of 6 sevenths divided by 3 sevenths? Of $\frac{6}{11}$ divided by $\frac{3}{11}$? Of $\frac{5}{11}$ divided by $\frac{2}{11}$?

3. If two fractions have the same denominator, how may their quotient be found?

4. Explain the process by which two fractions are changed to fractions having a common denominator.

The quotient of any two fractions may be found by changing them to fractions having a common denominator and then dividing the numerators of these fractions.

5. To divide $\frac{3}{4}$ by $\frac{2}{5}$ is the same as to multiply $\frac{3}{4}$ by what fraction?

The quotient of any two fractions may be found by inverting the divisor and multiplying.

6. State two ways by which the quotient of two fractions may be found.

7. Which of these processes is used in (1) below? In (2)?

(1) $\frac{2}{3} \div \frac{4}{5} = \frac{2 \times 5}{3 \times 5} \div \frac{3 \times 4}{3 \times 5} = \frac{\cancel{2} \times 5}{3 \times \cancel{4}_{2}} = \frac{5}{6}$.

(2) $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{\cancel{2} \times 5}{3 \times \cancel{4}_{2}} = \frac{5}{6}$

A reason for inverting the terms of the divisor and multiplying to find the quotient may be seen from the steps of (1) and (2). The second is the same as the first, only the fractions with common denominator are not written.

Written.

Divide:

8. $\frac{1}{2} \div \frac{1}{3}$. 9. $\frac{1}{4} \div \frac{2}{9}$. 10. $\frac{2}{5} \div \frac{1}{3}$. 11. $\frac{2}{3} \div \frac{1}{3}$.

12. $\frac{3}{4} \div \frac{1}{8}$. 13. $\frac{4}{5} \div \frac{5}{9}$. 14. $\frac{2}{7} \div \frac{2}{5}$. 15. $\frac{3}{8} \div \frac{6}{7}$.

Oral.

1. How many breadths of carpet $\frac{3}{4}$ of a yard wide are needed to cover a room 15 ft. wide?

2. How many breadths of plush $1\frac{1}{2}$ yd. wide are needed to make a curtain 45 ft. wide?

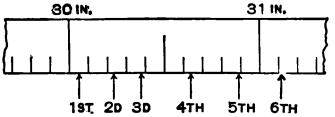
3. How many yards are there in a roll of carpet costing \$6 at $\frac{3}{4}$ of a dollar a yard?

4. Into how many pieces $\frac{5}{6}$ of a yard long can 15 yd. of ribbon be divided?

5. How many strips of carpeting $\frac{3}{4}$ yd. wide and 8 yd. long are required to cover a room 6 yd. wide and 8 yd. long?

6. $\frac{3}{5}$ of an acre of land is divided into two equal lots; what part of an acre is there in each lot?

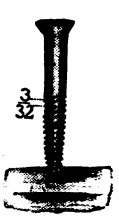
7. Into how many parts is each inch of this scale divided? What part of an inch is each division of the scale? How many hundredths of an inch is each of these divisions?



8. The arrows point to different readings of the scale. Thus, the reading of the first is 30.05 in.; of the second 30.23 in. Estimate and state the readings of the other arrows.

Written.

9. A certain screw is $3\frac{1}{4}$ in. long, the threads are $\frac{3}{32}$ in. apart. In one revolution the screw is driven a distance equal to that between two threads; how many revolutions must be made to drive the screw its whole length?



10. A screw is $2\frac{1}{2}$ in. long and the threads are $\frac{1}{8}$ in. apart; how many complete turns are required to drive it in?

11. A screw whose threads are $\frac{5}{64}$ in. apart is driven in by 28 revolutions; how long is the screw?

Oral.

1. What principle is employed when a fraction is changed to an equal fraction having a different denominator? Illustrate.

2. What is a fractional unit? Illustrate your answer.

3. How must fractions be expressed before they can be added or subtracted?

4. What is meant by the least common denominator of several fractions? Illustrate.

5. What is the least common denominator of $\frac{1}{3}$, $\frac{2}{5}$, $\frac{4}{7}$? Of $\frac{1}{12}$, $\frac{3}{8}$, $\frac{5}{6}$?

6. What is a mixed number? Explain two ways of adding mixed numbers. Illustrate each way.

7. Explain two ways of subtracting mixed numbers.

8. How may one fraction be multiplied by another?

9. What is the product of $\frac{7}{8}$ and $\frac{4}{5}$? Of $\frac{3}{5}$ and $\frac{5}{8}$? Of $\frac{4}{9}$ and $\frac{3}{5}$?

10. How may one fraction be divided by another? State another way.

Written.

Divide and test:

11. $\frac{3}{8} \div \frac{1}{5}$. 12. $\frac{5}{9} \div \frac{4}{10}$. 13. $\frac{24}{25} \div \frac{1}{100}$.

14. $\frac{22}{27} \div \frac{11}{3}$. 15. $\frac{13}{6} \div \frac{26}{3}$. 16. $\frac{57}{8} \div \frac{19}{4}$.

17. A rug $8\frac{1}{4}$ ft. by $10\frac{1}{2}$ ft. is placed on the floor of a room $10\frac{3}{4}$ ft. by 13 ft.; what is the area of the uncovered part of the floor?

18. What is the cost of a metal ceiling $18\frac{1}{2}$ yards long and $10\frac{1}{4}$ yd. wide at \$1.87 per sq. yd.?

19. A $17\frac{1}{2}$-acre field of corn yielded 48 bushels to the acre; at $62\frac{1}{2}¢$ per bu. what was the value of the yield? Various expenses amounted to $\frac{5}{16}$ of this sum; what was the profit?

20. If wheat had been grown in the field mentioned in Exercise 19, the profit would have been $\frac{1}{6}$ more; find the profit from wheat.

V

DENOMINATE NUMBERS

REDUCTION

NOTE.—All tables necessary for reduction of denominate numbers may be found on pages 229, 230.

1. How many times can a lamp holding 1 pint be filled from 4 gal. 3 qt. and 1 pt. of kerosene?

PLAN. 1. 1 gal. = — qt. 2. 4 × 4 qt. = — qt.
3. 16 qt. + 3 qt. = — qt. 4. 1 qt. = — pt.
5. 19 × 2 pt. = — pt. 6. 38 pt. + 1 pt. = — pt.
7. Therefore, the lamp can be filled — times.

A number of several denominations may be reduced to one of the lowest denomination by beginning with the highest and reducing step by step and adding like denominations whenever they occur.

2. How many inches are there in 7 yd. 1 ft. 7 in.?

3. Clarence picked 205 quarts of cherries; how many bushels and quarts did he pick?

PLAN. 1. 1 bu. = — qt.
2. 205 ÷ 32 = —? Hence, 205 qt. = 6 bu. 13 qt.

A number of one denomination may be reduced to the next higher denomination by dividing by the number of the given units which make one of the next denomination.

4. A milk wagon carried 125 pint bottles of milk; how many gallons, quarts, and pints was this?

Comparison of Weights. Precious metals are weighed by Troy weight.

The pound Troy contains 5,760 grains, or 12 oz. of 480 grains each. The ordinary pound (called *the* POUND *avoirdupois*) contains 7,000 grains.

5. How many pounds avoirdupois does a gold nugget weigh that weighs 39 oz. Troy?

Decimal Notation. The custom is growing of expressing quantities in one denomination only, using fractions if necessary. Decimal fractions are usually the most convenient.

Thus, 1.5 ft. would be used instead of 1 ft. 6 in., since .5 of 12 in. equals 6 in.

Written.

1. Express in hours and minutes: 1.3 hr.; 2.7 hr.; 4.9 hr.; 12.6 hr.; .5 hr.

2. Add the numbers as they are given in Exercise 1. Also when changed to hours and minutes. Which way is shorter?

3. Express in tons and pounds: 2.3 tons; 1.42 tons; 4.65 tons; 7.49 tons; 8.64 tons.

4. Add the numbers as given in Exercise 3. Also add when changed to tons and pounds. Which way is shorter?

5. How many hours, minutes, and seconds are there in .48 da.?

PLAN. 1. .48 × 24 hr. = 11.52 hr. 2. .52 × 60 min. = 31.2 min.
3. .2 × 60 sec. = — sec.
4. Therefore, .48 da. = 11 hr. 31 min. 12 sec.

6. How many yards, feet, and inches are there in 7.8 yd.? In 5.25 yd.?

7. How many square rods are there in 1.7 acre?

In the following give the results to the nearest hundredth:

8. 1 ft. is what part of a yard? Express decimally as parts of a yard: 1 ft.; 2 ft.; .4 ft.; $1\frac{1}{2}$ ft.; $\frac{3}{4}$ ft.; $\frac{1}{2}$ ft.

9. Express decimally 7 yd. 2 ft. and 11 in. as a number of yards.

PLAN. 1. 11 × $\frac{1}{12}$ ft. = .92 ft. 2. 2.92 × $\frac{1}{3}$ yd. = .97 yd.
3. 7 yd. + .97 yd. = — yd.
4. Therefore, 7 yd. 2 ft. 11 in. = 7.97 yd.

10. Express decimally, as a number of feet: 5 ft. 8 in.; 9 ft. 5 in.

Written.

1. Express decimally: $\frac{3}{4}$; $\frac{4}{5}$; $\frac{13}{20}$; $\frac{23}{25}$.

2. Express decimally: $6\frac{3}{4}$ ft.; $2\frac{4}{5}$ ft.; $7\frac{13}{20}$ ft.; $8\frac{23}{25}$ ft.

3. Add the numbers of Exercise 2 as given. Add the numbers expressed decimally. Which way is the shorter?

4. Add the numbers in the first column at the right. How many pounds and how many ounces besides is this? Add the second column, including the 1 lb. from the first.

120 lb.	12 oz.
119	4
118	8
358 lb.	8 oz.

5. A lot has a frontage of 18 yd. 2 ft. 6 in. and another has a frontage of 19 yd. 1 ft. 9 in.; find the frontage of the two lots.

6. The Empire State Express traveled from Rome to Albany in 2 hr. 49 min. 36 sec. and from Albany to New York City in 2 hr. 58 min. 50 sec.; how long was it in traveling from Rome to New York?

7. Subtract 17 hr. 54 min. from 25 hr. 36 min.

25 hr.	36 min.
17	54
7 hr.	42 min.

Add 1 hr. to the 36 min. of the right column and subtract 54 min. Add 1 hr. to 17 hr. of the left column and subtract the result from 25 hr.

8. An eclipse of the moon began at 8 hr. 38 min. 6 sec. P.M. and ended at 8 hr. 55 min. 51 sec. P.M.; how long did it last?

9. From a 10-gallon can of milk 21 quarts and 1 pint were sold; how much was left?

10. The best watches permit one to observe fifths of a second. A bicycle rider began a mile at 2 hr. 17 min. $23\frac{4}{5}$ sec. P.M. and ended it at 2 hr. 19 min. $56\frac{1}{5}$ sec. P.M. How long did it take him to ride the mile?

11. A rectangular room is 5 yd. $2\frac{1}{2}$ ft. long and 4 yd. $1\frac{1}{2}$ ft. wide; how many yards and feet of border are needed in papering the room?

Written.

1. A square lot measures 5 yd. 2 ft. 7 in. on a side; what is its perimeter?

Calculation.			*Steps.*
5 yd.	2 ft.	7 in.	1. 4 × 7 in. = 28 in. = 2 ft. 4 in.
		4	2. 4 × 2 ft. = 8 ft.
			3. 8 ft. + 2 ft. = 10 ft. = 3 yd. 1 ft.
23 yd.	1 ft.	4 in.	4. 4 × 5 yd. = 20 yd. 5. 20 yd. × 3 yd. = 23 yd.
			6. Therefore, the product is 23 yd. 1 ft. 4 in.

2. Express 5 yd. 2 ft. 7 in. as a number of feet and a fraction of a foot and multiply by 4. (This is a more common way of solving such problems.)

3. A wire fence 6 wires high is 20 yd. 2 ft. 9 in. long; how many feet of wire are needed for the fence? (Find the result in two ways.)

4. Express decimally to the nearest hundredth: 4 bu. 3 pk.; 2 bu. 1 pk.; 6 bu. 4 qt.; 8 bu. $1\frac{1}{2}$ pk.

5. Multiply the numbers as given in Exercise 4 by 10.

6. Multiply the decimal equivalents of the numbers of Exercise 4 by 10. Which way of multiplying is shorter?

7. How many 5-inch badges can be cut from 5 yd. 2 ft. 11 in. of ribbon?

PLAN. 1. 5 yd. 2 ft. 11 in. = 215 in. 2. 215 in. ÷ 5 in. = 43.
3. Therefore, 43 badges can be made.

8. 6 bu. 3 pk. 4 qt. of berries were divided equally among 4 customers; what quantity did each receive?

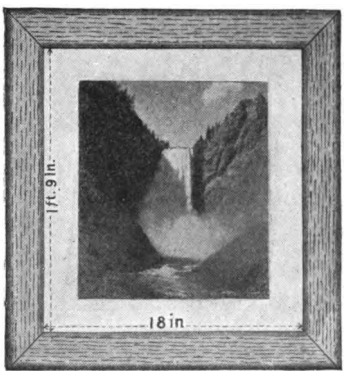
Calculation.	*Steps.*
	1. 6 bu. ÷ 4 = 1 bu. and 2 bu. remainder.
	2. 2 bu. + 3 pk. = 11 pk.
1 2 7	3. 11 pk. ÷ 4 = 2 pk. and 3 pk. remainder.
4)6 bu. 3 pk. 4 qt.	4. 3 pk. + 4 qt. = 28 qt. 5. 28 qt. ÷ 4 = 7 qt.
	6. Therefore, the quotient is 1 bu. 2 pk. 7 qt.

Divide by 5, both as the numbers stand and after expressing them decimally:

9. 207 lb. 8 oz. 10. 2 tons 600 lb. 11. 5 bu. 2 pk. 4 qt.

Written.

1. A bag of wheat weighed 118 lb. 8 oz., another 120 lb. 12 oz., and a third 119 lb. 4 oz.; what was their total weight?



2. A clerk sold $2\frac{1}{2}$ lb. of paper to one customer, 3 lb. 12 oz. to another, and 4 lb. 4 oz. to another; how many pounds did he sell in all?

3. How many feet of molding, $2\frac{1}{4}$ in. wide, are needed to make a frame for this picture?

4. The sides of a rectangular court measure 32 ft. 8 in. and 64 ft. 9 in.; what is the length of the railing surrounding the court?

5. From a ten-gallon can of gasoline a stove holding $7\frac{1}{2}$ qt. is filled; how much gasoline remains in the tank?

6. During four weeks it stormed 1 wk. 2 da. and 20 hr.; how many weeks, days, and hours were not stormy?

7. From a five-gallon can of kerosene were filled a tank holding 1 gal. 1 qt. and 1 pt., a lamp holding 1 qt. and 1 pt., and two other lamps holding $\frac{3}{4}$ of a pint each; how much kerosene was left in the can?

8. From a farm containing 140 acres and 40 sq. rd., a field containing 17 acres and 120 sq. rd. is sold; how many acres and square rods remain in the farm?

9. Multiply 3 t. 400 lb. by 20

10. Divide 420 yd. 2 ft. by 17

11. Add 3 wk. 20 hr. and 8 17

12. Subtract 10 gal. 1 qt. from 7 3

VI

FORM STUDY AND MEASUREMENT

FORM STUDY—TRIANGLES

Instruments: Rule and compasses

1. Draw a line 3 in. long, as A B in the figure. With the compasses draw an arc having A as center and radius equal to A B; also one with B as center and B A as radius. Draw straight lines to A and B from the intersection C of these arcs. Erase the arcs.

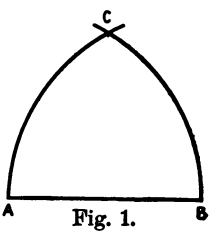
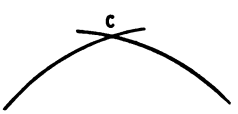


Fig. 1.

2. How do the sides of this triangle compare in length? Show that the method of construction makes them so.



3. Construct a triangle having each side 2 in. long.

Equilateral Triangle. A triangle whose sides are all equal is called an *equilateral triangle.*

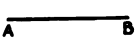


Fig. 2.

4. Draw a line 2 in. long, as A B in the figure. Draw an arc with radius 4 in. about A as center, and another with the same radius about B as center. Connect the intersection C with A and B by straight lines. Erase the arcs. Compare A C and B C.

Isosceles Triangle. A triangle having two sides equal is called an *isosceles triangle.*

5. Show that the construction in Exercise 4 makes the triangle isosceles.

6. Construct an isosceles triangle of base 3 in. and the other two sides each 5 in.

Instruments: Rule, compasses and right triangle or square

1. Construct an equilateral triangle of side 3 in. with altitudes as shown in Fig. 1.

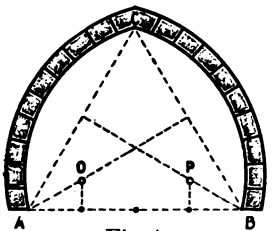


Fig. 1.

2. Divide the base into 4 equal parts. With the square draw perpendiculars through the outer points of division, cutting the altitudes in points, as O and P.

3. With O and P as centers construct arcs on B C and A C.

Explain from the following figures how to construct the arches:

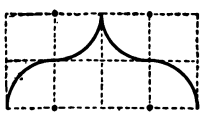


Fig. 2.

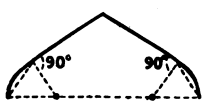


Fig. 3.

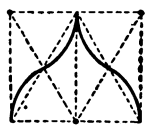


Fig. 4.

4. Construct an arch like Fig. 2; 3; 4. Arch 3 is called a *two-centered arch.* Why? What is arch 2 called? Arch 4?

5. Construct an equilateral triangle of side 3 in., as A B C in Fig. 5. With the rule divide each side into three equal parts.

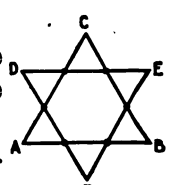


Fig. 5.

6. Through these points construct another equilateral triangle, as D E F. Connect A and E, D and B, F and C. If the figure is well drawn, these lines will pass through one point, P.

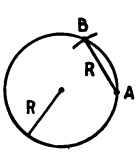


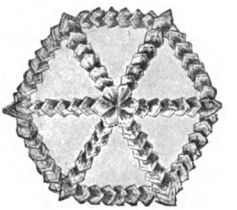
Fig. 6.

7. With P as a center and P A as a radius describe a circle. The circle should pass through A, F, B, E, C, D.

8. Draw any circle and a line A B equal to the radius, placed as in Fig. 6. Repeat the process with B as a center. Continue around the circle. What kind of a figure is formed?

1. What kind of triangles are used as bases for the designs of the previous page?

Bases for Designs. The circle, the square, the equilateral and the isosceles triangle, the hexagon and the octagon are common forms in architectural and decorative designs.



2. Anna designed this form for exhibiting handkerchiefs in a store window; show how to construct such a design.

3. If convenient, observe designs in which some of the forms mentioned are used, and describe them to the class.

4. Jennie made an apron from two handkerchiefs. Figure 1 shows how one handkerchief was cut, and Figure 2 shows how the two were made into an apron. Cut two

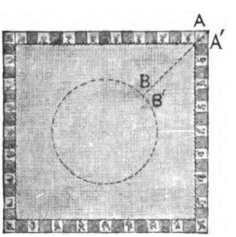


Fig. 1.

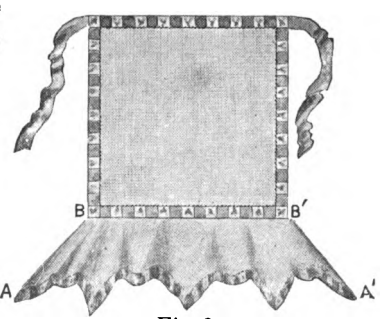


Fig. 2.

squares of side 11 in. from paper. Cut a circle out of the center of one so that its length shall be as nearly equal to the side of the square as possible. What must the radius of the circle be? Cut along the diagonal A B, as shown in Figure 1. Place them together as in Figure 2.

5. Place two equilateral triangles together so as to form a parallelogram; place three so as to form a trapezoid; place four so as to form a triangle.

1. Draw a right triangle having one angle 45°. What is the ratio between the sides including the right angle? What is the length of a in Figure 1?

2. Draw a right triangle having the angle A = 60°. Measure b and a, and find the ratio $\frac{a}{b}$ to the nearest tenth.

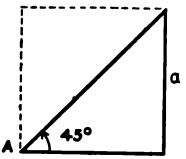


Fig. 1.

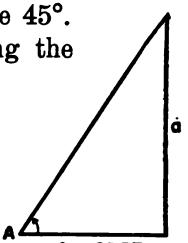


Fig. 2.

3. If the angle is made greater than 45°, as A in Figure 2, and the base remains the same, how will a change? How will the ratio of a to b change?

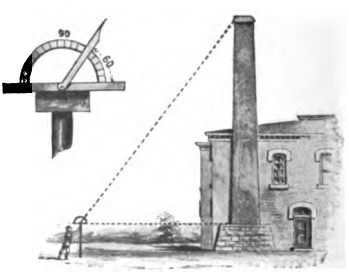
ANGLE.	RATIO OF a TO b.
10°	0.18
20	0.36
30	0.58
40	0.84
45	1.00
50	1.19
60	1.73
70	2.25
80	5.67

4. If $b = 20$ and the ratio $\frac{a}{b}$ is 2, what is the length of a?

The table gives the ratio of a to b for various angles from 10° to 80°.

5. Using the table, find b when the angle is 10° and a is 25 ft.

6. Make 5 other problems requiring use of the table.



7. Clifford arranged a protractor and two narrow pieces of wood, as shown in the picture. While holding the lower one horizontally he made the upper arm point to the top of a chimney. The angle thus made was 60° and the horizontal distance from the chimney was 40 ft.; what was the height of the chimney?

1. By counting spaces find the length of the line A B; of the line B C; what is the ratio of these two lines?

2. Similarly find the lengths and the ratio of the lines A′ B′ and B′ C′.

3. How do these ratios compare?

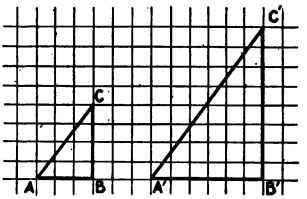


Fig. 1.

4. Draw a right-angled triangle so that A B = 5 and B C = 12. Draw another so that A B = 10 and B C = 24. Measure A C and find the ratio of A C to B C in each case. How do they compare? (If available, use paper ruled in squares.)

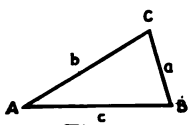


Fig. 2.

5. Draw a triangle with sides $a = 1\frac{1}{2}$ in.; $b = 2\frac{1}{2}$ in.; and $c = 3$ in.; draw another, A′ B′ C′, with sides twice as long.

6. If a protractor is available measure and compare angles A and A′; B and B′; C and C′.

Similar Triangles. Triangles whose corresponding sides have the same ratio are called *similar triangles.* Their corresponding angles are equal.

7. The ratio of two sides of a triangle is 10. What is the ratio of the corresponding sides of any similar triangle?

8. The triangles of Fig. 3 are similar. To what ratio is the ratio $\frac{H}{6}$ equal? What does H equal? Find C.

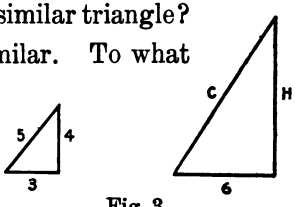


Fig. 3.

9. Miles observed that a telegraph pole cast a shadow of 15 ft. when a 5-foot stick held vertically cast one of 2.5 ft. He determined that the pole was 30 ft. high. How could he find this? Draw the similar triangles.

10. If convenient, find other heights in the same way.

1. According to the measurements shown in Fig. 1, what is the length of the line A B?

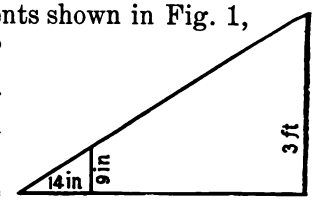


Fig. 1.

2. A tower stands on a horizontal plane. A line is measured from the base of the tower to a point 80 ft. distant. At this point, a line to the top of the tower makes an angle of 20° with the line just measured. What is the height of the tower? (Use table, page 48.) What is the height of a tower for which the angle is 30°? 45°? 60°? 80°?

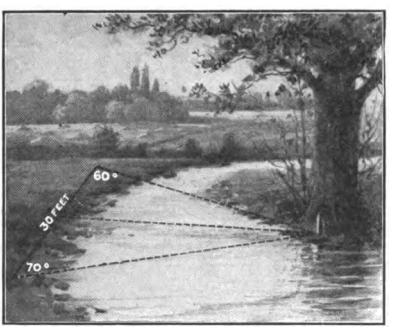


Fig. 2.

To find the width of a creek, two boys made the measurements indicated in Fig. 2.

3. With the aid of a protractor make a drawing of the lines shown in Fig. 2, using 1 in. for 10 ft. Measure the length of the altitude (the middle dotted line in the figure), and find the distance from the tree to the opposite bank of the creek.

4. A map is drawn to the scale indicated in Fig. 3, that is, 1 inch corresponds to 75 miles. What is the distance between two places, represented by dots on the map which are $9\frac{1}{2}$ in. apart?

Fig. 3.

5. How far apart on the same map are the dots which represent two places 840 miles apart in fact?

6. Three cities are connected with one another by three roads forming an equilateral triangle; an altitude of the triangle is 5 mi.; by use of the table, page 48, find the distance between each pair of cities.

Instruments: Rule, compasses and triangle

1. Construct an equilateral triangle of sides 3 in.

2. Bisect the sides of the triangle and connect these points by lines, as shown in Figure 1.

3. Fold the triangle along these lines and bring the corners together.

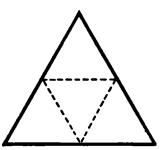


Fig. 1.

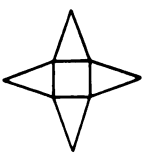


Fig. 2.

4. Construct a square of side 2 in. On each side of the square construct an isosceles triangle of side 3 in. (See Fig. 2.)

5. Fold so as to bring the vertices of the triangles together.

Pyramids. The forms made in Exercises 3 and 5 are called *pyramids*.

The base, or bottom, of a pyramid may have any number of sides. Pyramids are called *triangular*, *square*, and the like, according to the shape of the base. What kind of figures are the other faces of a pyramid?

Fig. 3.

Fig. 4.

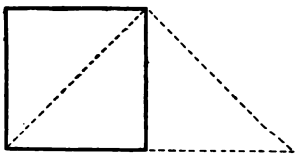
John wrapped a piece of paper around a square pyramid and trimmed it off even with the base. He also wrapped a piece of paper around a square prism with base and altitude equal to those of the pyramid, leaving one end open and trimmed off even with the solid. After pasting the margins so as to hold the forms firmly in shape, he filled the paper pyramid with sand and then poured the sand into the paper prism. By doing this three times, he filled the prism.

6. If convenient, perform an experiment similar to this.

7. The volume of the pyramid was how many times as great as that of the prism? How did their bases and altitudes compare? How is the volume of a square prism found?

8. According to the result of John's experiment, how may the volume of a square pyramid be found?

Chester drew a square 1 in. on a side and a triangle of equal area, as shown in the figure. He modeled from clay two pyramids of equal altitudes, one having the square for the base and the other the triangle. The weights of the two pyramids were the same. Chester concluded that the volume of a triangular pyramid may be calculated in the same way as that of a square pyramid.

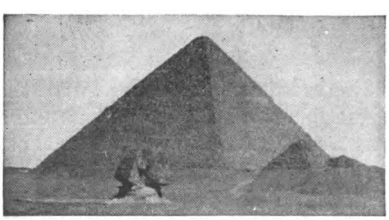


1. If Chester's conclusion was correct, how may the volume of a triangular pyramid be calculated when the altitude and the area of the base are known?

It can be proved that: **The volume of any pyramid, no matter what its base, is $\frac{1}{3}$ of the product of the area of the base and the length of the altitude.**

2. Find the volume of a triangular pyramid, the area of whose base is 36 sq. in. and whose altitude is 8 in.

3. The pyramid of Cheops, shown in the picture, stands on a square base 746 ft. on a side; what is the area of the base? The altitude of this pyramid is 480 ft. Find its volume.



4. What kind of figures are the lateral sides of a pyramid? How is the area of a triangle computed? Find the lateral area (surface, excluding the base) of the pyramid of Cheops (altitude of side 608 ft.).

5. Find the numbers to fill the blanks:

ALTITUDE OF PYRAMID.	BASE (Rectangular).	VOLUME.
10 ft.	4 ft. by 5 ft.	———
16 yd.	—— sq. yd.	1,728 cu. yd.
——	10 in. by 9.5 in.	1,187.5 cu. in.

Oral.

1. What is an equilateral triangle? An isosceles triangle?

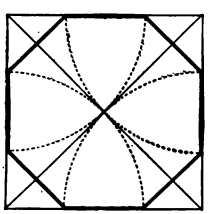
2. Explain how to construct with the aid of compasses an equilateral triangle of side 4 in.; also explain how to construct an isosceles triangle of base 3 in. and sides 4 in.

3. If a circle is drawn through the vertices of a regular hexagon, how does the length of each side of the hexagon compare with the length of the radius of the circle?

4. Name some object in the form of a pyramid. What shape may the base of a pyramid have? The other sides?

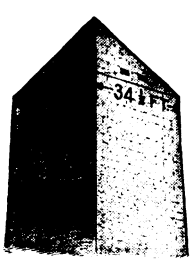
5. How is the volume of a pyramid computed? What is the volume of a pyramid of base 16 sq. in. and altitude $5\frac{1}{2}$ in.?

Written.



6. Draw a square of side 2 in. Draw its diagonals. With the corners of the square as centers describe arcs as shown in the figure. Complete the figure.

7. A pyramid is 50 ft. square and 75 ft. high. Find its volume.



8. The Washington monument is crowned with a square pyramid as shown in the picture. The vertex of the pyramid is 25 ft. above its base. Find its volume.

9. Find the altitude of a pyramid of which the volume is 84 cu. ft. and the base is a square 6 ft. on a side.

Find the missing numbers in this table concerning pyramids:

	10.	11.	12.	13.	14.	15.
Volume (cu. ft.)	500	250	—	600	—	112
Altitude (ft.)	—	30	25	20	15	32
Area of Base (sq. ft.)	50	—	40	—	49	—

VII

EQUATIONS

LITERAL NOTATION

Oral.

1. If t represents the age of a man now, what represents his age 10 years from now? y years from now? 12 years ago? n years ago?

2. If A earns b dollars in a week and spends c dollars in the same time, what represents the amount that he saves?

3. A piece of merchandise is bought for d dollars and sold at \$10 profit; what stands for the selling price?

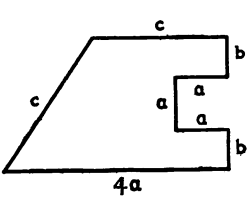
4. A watch was sold for \$25 and there was \$$d$ profit; what stands for the cost of the watch?

5. A box and contents weigh 140 lb. The box weighs p pounds. What stands for the weight of the contents?

6. If one book weighs p pounds and another q pounds, what is their combined weight?

7. At x cents a peck, what represents the cost of 1 bushel?

8. At b cents an ounce, what represents the cost of 3 oz.? Of 1 lb.? Of 6 lb.? Of p lb.?



9. If 5 represents the side of a square, what represents its perimeter?

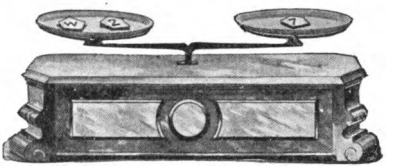
10. How many dimes are there in 5 dollars? In d dollars? In $3x$ dollars?

11. Find the perimeter of the figure at the left.

12. How many ounces are there in 3 pounds? In x pounds? In $5a$ pounds?

Oral.

1. How heavy must the weight, w, be in order that the two pans may balance each other?



The condition of balancing may be represented by $w + 2 = 7$.

State the condition of balancing for each of the following:

2. **3.** **4.**

5. The picture shows a balance made by suspending a square piece of board from each end of a yardstick. A

quart measure filled with wheat is placed on one board and enough weights to balance it on the other.

If the cup when empty weighs 4 oz., what does the wheat in the cup weigh? If the wheat is poured loosely on one side of the balance, how many ounces are needed on the other side to balance it?

The solution may be outlined thus:

1. Let w be the number of ounces in the weight of the wheat.
2. Then, $w + 4 =$ the number of ounces in the weight of the wheat and the cup.
3. Then, $w + 4 = 34$. Why? 4. Hence, $w = 30$. Why?

Written.

6. A half-peck measure weighs $\frac{3}{4}$ lb. When filled with wheat, it weighs $8\frac{1}{4}$ lb. Find the weight of $\frac{1}{2}$ pk. of wheat. Express the solution in the form given in Exercise **5**.

7. Find the value of w in Exercises **1**, **2**, **3**, and **4**.

Written.

The table shows the results found by testing the weights of various grains by the plan given on page 55 :

GRAIN.	Weight of quart measure.	Total weight.	Weight of 1 quart of grain.	Weight of 1 bushel of grain.
Oats.....	4 oz.	$1\frac{1}{4}$ lb.	——	——
Rye.....	4 "	2 "	——	——
Barley.....	4 "	$1\frac{3}{4}$ "	——	——
Corn (shelled)..	8 "	$2\frac{1}{4}$ "	——	——
Buckwheat....	6 "	$1\frac{7}{8}$ "	——	——

1. Complete the table, using the form of solution given on the preceding page.

2. If convenient, make similar experiments. Are equal amounts of different samples of grain apt to be equal in weight?

The results for the last column of the above table are the weights fixed by law in various states as the minimum weight of 1 bu. of each kind of grain.

3. An empty bag weighs $1\frac{1}{2}$ lb. When filled with 2 bu. of potatoes it weighs $121\frac{1}{2}$ lb.; what is the weight of 1 bu. of potatoes?

PLAN. 1. Let w = the number of pounds in the weight of 1 bu. of potatoes.

2. Then, $2w + 1\frac{1}{2} = 121\frac{1}{2}$. Why?

3. $2w = 120$.

4. $w =$ ——.

4. A 2-bushel bag, which weighs 8 oz. when empty, weighs $40\frac{1}{2}$ lb. when filled with bran; what is the weight of 1 bu. of bran?

5. A bushel measure weighing 3 lb., weighs 63 lb. when filled with wheat; what is the weight of 10 bu. of wheat?

6. An empty car weighs 30,000 lb.; when loaded with 300 bu. of peas it weighs 48,000 lb.; what is the weight of the peas per bushel?

Oral.

What value must be given to n *in turn, that each of the following expressions may have the value 24?*

1. $2n$. 2. $6n$. 3. $\frac{1}{2}n$.
4. $3n + 6$. 5. $11n + 2$. 6. $5n + 4$.

What values must be given to y *in turn that each of the following expressions may be true?*

7. $9y = 18$. 8. $5y + 2 = 22$.
9. $3y = 6$. 10. $4y + 6 = 46$.
11. $7y = 35$. 12. $8y + 12 = 20$.
13. $15y = 5$. 14. $9y + 5 = 50$.

Equations. Expressions of equality are called *equations.*

The equation $3x = 15$ is said to be *satisfied* by the number 5, because 3 times 5 = 15.

It is not satisfied by the number 6, because 3 times 6 does not equal 15.

Find by inspection which of the following equations are satisfied by 1, which by 2, and which by 3:

15. $4x = 8$. 16. $7y = 21$. 17. $10y = 20$.
18. $23y = 23$. 19. $16h = 48$. 20. $11x = 22$.
21. $4f = 20$. 22. $5x + 1 = 6$. 23. $2y - 1 = 3$.
24. $10r - 3 = 27$. 25. $4m + 2 = 10$. 26. $n - 1 = 2$.

Root. Any number which satisfies an equation is called a *root* of that equation. The process of finding the root is called *solving* the equation.

Read each of the following equations and give the root at sight:

27. $11x = 22$. 28. $6y = 18$. 29. $9r = 36$.
30. $16y = 48$. 31. $15r = 75$. 32. $3p = 33$.
33. $9y = 12$. 34. $20x = 15$. 35. $5x = 4$.
36. $4m = 6$. 37. $3y = 1$. 38. $7y = 12$.
39. $5z = 2$. 40. $18z = 6$. 41. $6y = 4$.
42. $10x = 15$. 43. $44r = 11$. 44. $9s = 45$.

Written.

Problems may often be stated most clearly in the form of equations, using some letter to represent the unknown quantity.

State in the form of equations and find the numbers required:

1. What number increased by 3 equals 12?

PLAN. 1. Let n represent the number.
2. Then $n + 3 = 12$.
3. Hence, $n =$ —. Why?

2. What number increased by 6 equals 16?

3. What number must be doubled and the result increased by 12 to produce 28?

PLAN. 1. Let n stand for the number. 2. Then $2n + 12 = 28$.
3. Hence, $2n = 16$. Why? 4. Hence, $n =$ —.

4. To what number must 16 be added to produce 78?

5. A salesman sold twice as many hats on Tuesday as on Monday, and six more on Wednesday than on Tuesday. On Wednesday he sold 16. How many did he sell on Monday?

6. Mary thought of a number; doubled it; added 12 and obtained 58. Of what number did she think?

The equation is $2n + 12 = 58$. Why?

7. In 6 years John will be 4 times as old as James is now. James is now 4 years old; how old is John?

The equation is $x + 6 = 16$. Why?

8. If Mr. Brown's annual salary was increased by $\frac{1}{5}$ of itself, it would lack \$600 of being \$2,000; what is his salary?

9. A train leaves Chicago for Omaha, 491 mi. distant, at 8 P.M. It runs at the rate of 42 mi. an hour. When is it 183 mi. from Omaha?

PLAN. 1. When the train is 183 mi. from Omaha, it is — mi. from Chicago.
2. Let $x =$ the number of hours from the time the train leaves Chicago until it is 183 mi. from Omaha.
3. $42x =$ — mi. 4. $x =$ —. 5. $7\frac{1}{3}$ hr. after 8 P.M. is —.

Oral.

1. William weighs $7a$ pounds and Frank weighs $9a$ pounds, how much do they weigh together?

State:

2. The number that is 4 greater than n.

3. The number that is a greater than n.

4. The number that is 3 less than n.

5. The number that is b less than n.

6. If one boy can carry 12 books, how many books can 4 boys of equal strength carry? If one boy can carry n books, how many books can b boys carry?

7. If oranges cost 20 cents a dozen, what will 5 dozen cost? What will d dozen cost at c cents a dozen?

8. If 3 men can dig a ditch in 4 days, how long will it take one man to dig the same ditch? If x men can dig it in d days, how long will it take one man?

9. What is an equation? Illustrate your answer.

10. What is meant by solving an equation?

11. What is a root? What is meant by satisfying an equation? Illustrate.

Written.

Solve the following equations:

12. $8x + 5 = 9$.

13. $15s + 2 = 12$.

14. $12x + 1 = 4$.

15. $11h + 7 = 29$.

16. $39g + 7 = 85$.

17. $5f + 11 = 15$.

18. $7r + 3 = 4$.

19. $1 + 8x = 3$.

20. $12 + 3x = 17$.

21. $\frac{1}{2}x + 2 = \frac{7}{3}$.

22. George had 78¢ and earned some more. He then had 3 times as much as he had earned; how much had he earned?

23. A farmer expected to reap 10 times as much wheat as he sowed. He reaped 347 bu., which was 53 bu. less than he expected; how much did he sow?

VIII

PERCENTAGE AND ITS APPLICATIONS

PERCENTAGE OF NUMBERS

Oral.

1. 37% by weight of a certain grade of ingrain carpet is cotton and the rest is wool; how many pounds per 100 lb. of this carpet are cotton? Wool?

2. What is the meaning of per cent? Illustrate.

3. The duty imposed by the United States Government upon Oriental rugs is 40%; how many dollars are collected on each $100 worth of rugs?

4. A supply house offered a 5% discount on orders amounting to $100 or more. What would an order amounting to $100 cost the customer? One amounting to $500?

5. The timber land of a certain farm was 25% of the whole; what per cent was cleared?

6. How many hundredths of the farm of Exercise 5 were timbered? Cleared? What part was timbered? Cleared?

7. $\frac{1}{4}$ of a number is what per cent of the number?

8. $\frac{1}{3}$ of a number is what per cent of the number?

9. $\frac{2}{3}$ of a number is what per cent of the number?

10. $\frac{3}{5}$ of a number is what per cent of the number?

11. $\frac{5}{8}$ of a number is what per cent of the number?

12. 5 is what part of 20? 5 is how many per cent of 20?

To find what per cent one number is of another, divide the first by the second. Express the quotient in hundredths. The number of hundredths is the number per cent.

13. 15 is what per cent of 30? Of 60? Of 75?

14. 45 is what per cent of 90? Of 900? Of 9?

15. 3 is $\frac{1}{8}$ of what number? 3 is —% of 9?

Oral.

1. Ethel answered correctly 9 questions in a list of 10; what per cent did she answer correctly? What per cent did she miss?

2. Roland solved 7 problems in a list of 10; what per cent did he solve? What per cent did he fail to solve?

3. A spelling test paper containing 100 words was marked 83%; how many words were spelled correctly? How many were spelled incorrectly?

4. An arithmetic test paper containing 5 problems of equal rank was marked 80%. Each problem was marked right or wrong. How many were right? How many were wrong?

5. The attendance of 25 pupils for a month of 20 school days is equal to how many days' attendance of one pupil?

6. There were absences among the pupils of Exercise 5 amounting to 5 days during the month; what was the percentage of absence? Of attendance?

7. There was a possible attendance of 2,000 days for a class in one term. There were absences amounting to 40 days; what was the percentage of absence? Of attendance?

8. The window surface in a schoolroom should be $\frac{1}{5}$ of the floor surface; what per cent is this?

9. According to Exercise 8, how many square feet of window surface should a room 20 ft. square have? A room 30 ft. by 50 ft.? One 20 ft. by 25 ft.?

10. What is the floor surface of a properly lighted room whose window surface is 40 sq. ft.? 100 sq. ft.?

11. 40% of the air in a schoolroom was changed by ventilation in 12 minutes. At the same rate in how many minutes would it all be changed? The room contained 4,000 cu. ft.; how many cubic feet were changed in 12 min.? In 30 minutes? In 1 hr.?

Written.

1. In a recent month the employees of the Chicago post-office handled 48,000,000 pieces of mail matter. Of these 100,000 were addressed defectively. How many pieces of mail matter were addressed defectively per million? Per thousand? Per hundred? Per cent?

2. What part of the whole was addressed defectively?

3. Of the defectively addressed mail the names of 85,000 addresses were worked out sufficiently to permit delivery as intended by the writer. How many of the defectively addressed pieces were correctly delivered per thousand? Per hundred? Per cent?

4. What part of the defectively addressed matter could not be delivered?

5. In the same month (of 30 days) the clerks of the Chicago post office made 503 recorded mistakes and the general public 675,717 recorded mistakes in addressing and stamping mail matter; what was the daily average for each? How many mistakes were made by the public for each mistake of the clerks?

6. The steamship Teutonic brought from Europe in one trip 527,270 letters; estimating their average weight at $\frac{1}{2}$ oz., how many tons of letters were there?

7. The weight of the letters as found in Exercise 6 was about 40% of the weight of the second class matter brought by the same mail; how many tons of second class matter were there?

8. Allowing 5 in. for the length of each envelope, how many feet would the letters of this mail reach, if placed end to end?

9. 870 bags of mail were received from the steamer at the rate of 3 bags in two minutes, how long did it take to unload them?

Oral.

1. A merchant bought cloth at \$1 per yard and sold it at \$1.25; how much did he gain per yard? 25¢ is what per cent of \$1? What was the percentage of gain?

2. A grocer bought berries at 10¢ per quart and sold them at 8¢; how much did he lose per quart? 2¢ is what per cent of 10¢? What was the percentage of loss?

Written.

Find the missing numbers:

ARTICLES.	Cost per Pound.	Selling Price per Pound.	Part of Cost Gained.	Gain Per Cent of Cost.
3. Nails.....	\$.02	\$.04	—	—
4. Butter.....	.20	.26	—	—
5. Sugar	.04½	.05	—	—
6. Cinnamon.....	.90	—	$\frac{1}{5}$	—
7. Ham.....	.15	—	—	20
8. Cheese.....	—	.25	—	$8\frac{1}{3}$
9. Nuts.....	—	.15	$\frac{1}{6}$	—
10. Sausage.....	—	.12	$\frac{1}{3}$	—
11. Veal cutlets...	—	.20	$\frac{1}{7}$	—

12. A bicycle costing \$50 was sold at a gain of 40%; what was the selling price?

13. An automobile is sold for \$650 at a gain of 30%; if c represents the cost, $1.3\,c = \$650$ represents the cost plus the gain or selling price; what is the cost?

Find the cost in each case:

	14.	15.	16.	17.
Selling price:	\$240	\$390	\$170	\$1,400
Gain or loss:	20% (gain)	30% (gain)	15% (loss)	40% (gain)

18. A fruit dealer stored 1,000 barrels of apples for 6 mo. at 5¢ a barrel per month. The storage charge was 15% of the selling price; for how much did the 1,000 barrels sell?

19. A farmer stored 500 tons of cabbage for 3 mo. at 5¢ a ton per month. The storage charge was 5% of the selling price; for how much did the 500 tons sell?

Commission. Services in buying or selling for others are often paid at a fixed rate per cent of the value of the article sold. Such payment is called *commission.*

Commission Merchant. An agent who handles and sells commodities on commission is called a *commission merchant.*

Consignor. A person who sends articles to a commission merchant is called a *shipper*, *consignor*, or *principal.*

Consignment. The merchandise sent is called a *consignment.*

Broker. An agent who arranges for purchases or sales without actually receiving and delivering the goods is called a *broker.*

Thus, when a commission merchant sells a carload of apples he actually receives and delivers the apples; but a broker in Chicago representing a wheat owner in Minneapolis may sell a carload of wheat in the Chicago market to another broker who represents a miller in Cincinnati. The wheat would be shipped from Minneapolis direct to Cincinnati.

Oral.

1. A broker received $\frac{1}{8}¢$ per bushel for buying ten carloads of wheat containing 400 bu. each; what was his brokerage?

2. A commission merchant sold a consignment of peaches for $200, charging 5%; what was his commission?

Find the commission at the rate given on each consignment:

	CONSIGNMENT.	RATE.
3.	500 crates of berries sold at $1 a crate	10%
4.	300 tons of cabbage sold at $1 a ton	8%
5.	50 crates of eggs at $5 a crate	10%
6.	200 bbl. of apples sold at $2 a barrel	10%
7.	100 forty-pound tubs of butter sold at 20¢ a pound.	5%
8.	40 dressed lambs, 90 lb. each, sold at 10¢ a pound..	5%

9. A broker bought 8,000 bu. of wheat; what was his brokerage at $\frac{1}{8}¢$ per bu.?

10. A broker bought 7,000 bu. of corn and sold 5,000 bu. of rye; what was his brokerage at $\frac{1}{8}¢$ per bu.?

Oral.

1. A dry goods merchant marked some lawn goods 30¢ a yard and later in the season offered them for 25¢; what was the reduction per yard?

2. The reduction in Exercise 1 was what per cent of the original marked price?

Commercial Discount. A reduction made in the price of goods is called a *discount.*

Discounts are made for various reasons, as for prompt payment by the purchaser, or on account of damage to the goods, or deterioration.

Price Lists. For various commodities, standard prices are fixed and published in *price lists.* These prices are higher than the usual selling price, and the selling price is specified by the discount allowed from the *list price.* The price after the discount is deducted is called the *net price.*

3. A discount of 10% is offered on the following prices; find the net prices:

Range	$40	Saw	$3.50
Refrigerator	16	Keg of nails	3.00
Gasoline stove	22	200 feet of rope	16.00
Water heater	12	100 lb. of steel rod	5.00
Bath tub	30	2 dozen door fixtures	27.00

4. A supply house offers a discount of 10% on all orders amounting to $100 or more and a further discount for cash of 5% of the amount left after the first discount is deducted; what was the net price on cash orders amounting to $100? $200? $120? $300? $500?

Written.

5. A firm offers a discount of 20% on the following prices; find the cost of 5 pennants of each kind:

Flags and Pennants.	Silk—one color.	Silk—two colors.	Cashmere
12 in. by 18 in.	$0.85	$1.15	$0.70
16 in. by 24 in.	1.00	1.50	0.85
24 in. by 36 in.	1.35	2.15	1.15

6. If convenient, make similar problems from price lists.

Taxes. In towns, counties and states, the expense of government, the cost of public enterprises and charities are met chiefly by collecting from property owners a certain amount on each dollar of property valuation. Sums collected for these purposes are called *taxes.*

Rate. The *rate* of taxation is expressed by a rate per cent or by an amount on each dollar.

Thus, $\frac{1}{2}\%$, or 5 mills, means a tax of 5 mills on every dollar.

Assessed Value. Taxes are usually not computed on the full value of the property. The value taken as a basis of taxation is called the *assessed value.*

Thus, if a house and lot worth $1,600 is valued for taxation at $1,200, it is said to be *assessed* at $\frac{3}{4}$ of its value.

Written.

1. The following is the annual budget voted by a village board of trustees; find the total appropriation:

Contingent fund.....	$10,000	Fourth district, highway fund.....	$700
Street lighting	3,700	Fire department.....	3,500
First district, highway fund.....	1,200	Poor fund.....	1,700
Second district, highway fund.....	1,200	Park fund.....	900
Third district, highway fund.....	1,500	Bridge fund.....	4,000
		Sidewalk fund.....	2,500
		Library fund.....	700

2. What per cent of the total was the contingent fund? The poor fund?

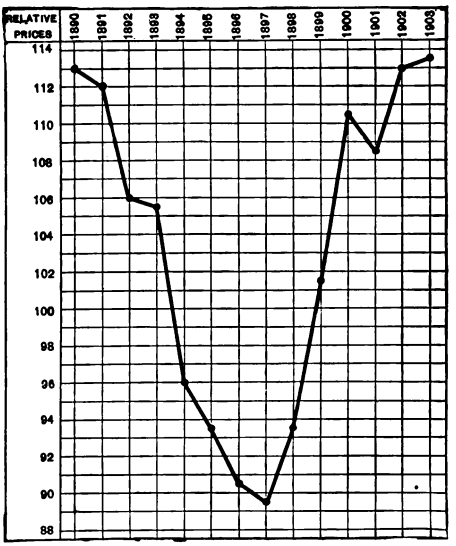
3. Make ten other problems about these appropriations.

4. A house and lot valued at $2,400 is assessed at $\frac{2}{3}$ of its value; how many dollars is this? What is the tax at $\frac{7}{8}\%$?

5. A property owner's tax is $30 on an assessment of $1,500; what is the rate of taxation?

6. $\frac{3}{4}$ of the budget mentioned in Exercise 1 was raised by direct taxation, the rate being $1\frac{1}{2}\%$; find the total assessment on which the tax was based.

The diagram shows, according to the Bureau of Labor, that what \$113 would buy in 1890 could be bought for \$112 in 1891, or for \$106 in 1892, or for \$96 in 1894, and so on.



Oral.

1. For what could the same amount be bought in 1896? In 1897? In 1899? In 1900?

2. The amount that could be bought for \$1.06 in 1892 cost how much in 1894? In 1897? In 1900?

3. Compare the prices in 1891 with those of 1901.

4. The highest price is how much above the lowest?

5. The 100 line represents the average price from 1890 to 1899; what per cent is the price for each year above or below this average?

6. In which of these years were the relative prices above the average? Below the average?

Written.

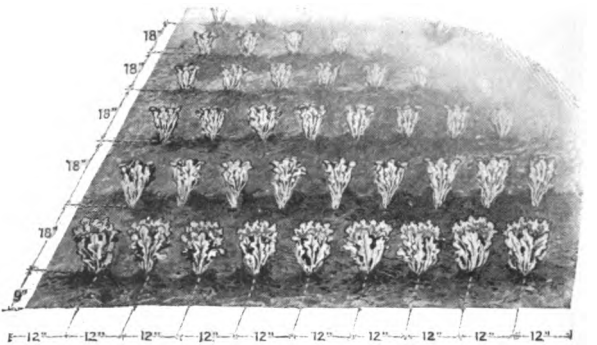
7. 7 is what per cent of 113? What was the percentage of fall in prices from 1890 to 1892?

8. 12 is what per cent of $89\frac{1}{2}$? What was the percentage of rise in prices from 1897 to 1899? From 1899 to 1900?

9. Make and solve 5 other problems about this diagram.

Written.

1. According to the usual method of growing beets, the rows are 18 in. apart and the plants 12 in. Allowing 12 in. for margins at each end and 9 in. at each side, how many



plants can be grown on a field 110 yd. long and 44 yd. wide when the rows run the long way? How many acres are there in the field?

2. If 5 lb. of seed per acre are required, what is the cost of the seed for 8 acres at 16¢ a pound?

3. Find the cost of growing 5 acres of beets when the cost per acre of preparing the soil is $1.55, of seed and planting $3.25, and of cultivating and care, $31.

4. If it costs $230.20 to grow and harvest a 5-acre crop of beets, what is the cost of harvesting?

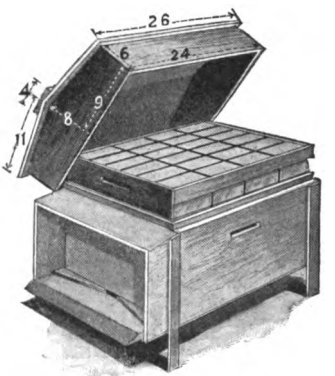
5. If the beets from 1 acre sell for $56.04 at $4.38 a ton, what is the yield from each acre?

6. At this rate how many tons were grown on the 5 acres? What was the total profit? What was the profit on each acre? What was the percentage of profit?

7. If the total receipts for sugar, molasses, and pulp is $11.10 per ton of beets, and if the total cost of these products is $7.84, what is the percentage of profit to the manufacturer?

Written.

1. How many board feet of lumber would it take to make a cover for this beehive, allowing 1 board foot for waste? (The dimensions are given in inches.)



2. The crate shown in the top of the hive contains sections for 1 lb. of honey each; how many pounds does this crate hold? If a colony of bees fills the crate twice in a season, what is the value of their season's product at 12¢ a pound?

3. If from the same product 40 lb. of strained honey could be extracted and sold for 15¢ a pound, which is the more profitable, comb honey or extracted honey? The difference is what per cent of the smaller amount?

4. Taking the average production of honey to be 35 lb. a season and the selling price of the honey to be 9¢, what is the value of the product of an apiary of 200 colonies? If it takes $\frac{1}{3}$ of this amount to meet incidental expenses and to pay the interest on the investment, what is the profit of the apiary for one season? What per cent of the income is profit?

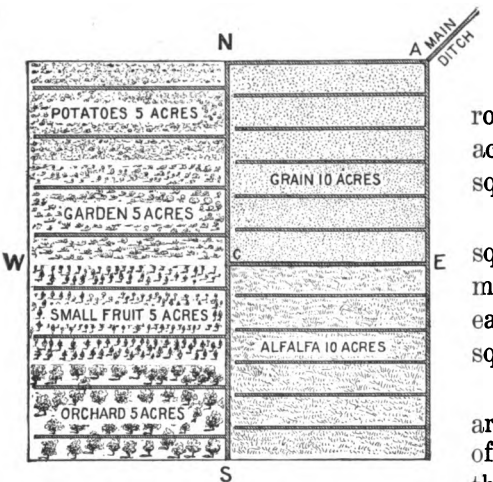
5. The area over which a colony of bees generally ranges is equal to a square 5 miles on a side; how many acres is this?

6. A good bee pasture should average 1 honey-bearing plant per square foot; how many honey-bearing plants are there in 25 sq. mi. of such a pasture?

7. If only $\frac{1}{4}$ of the number of plants are in blossom at one time, what per cent of the pasture is available?

Oral.

1. The picture shows a plan for irrigating a square farm. How many acres does the farm contain?



2. How many square rods are there in 10 acres? (1 acre = 160 sq. rd.)

3. What is the square of 40? How many rods are there in each side of a 10-acre square?

4. How many rods are there in each side of the square farm of the picture?

Written.

5. The double shaded lines, as N S for example, represent irrigation ditches; taking each ditch running east and west to be 40 rd. long, how many rods of ditches are there in all?

6. Indicate the number of sq. ft. in 1 sq. rd. (1 rd. = $16\frac{1}{2}$ ft.) Find the number of square feet in 1 acre.

7. If the ditches on the alfalfa field deliver 2 cu. ft. a second, how many hours will it take to deliver a volume of water equal to a lake 6 in. deep over the whole field?

8. If, in excavating the main ditch and the laterals for a 10-acre farm, 250 cu. yd. of earth were removed at a cost of 35¢ a cu. yd., what did the excavating cost?

9. If the surveying cost $12, the boxes and gates $125, what was the total cost of the system?

10. If the annual cost of maintenance and operation is 35% of the first cost, how many dollars is this per acre?

In localities where irrigation from a stream is necessary, the natural flow of the stream supplying the water is usually insufficient for a part of the year, while at other times there is a surplus. In this case the surplus water is collected and stored in reservoirs to make up the deficiency during the rest of the year.

Written.

1. Logan River, Utah, has an annual discharge of 369,000 acre feet. (An acre foot is the amount of water necessary to cover 1 acre to the depth of 1 foot.) Of this amount 57% is used when discharged; how many acre feet are so used?

2. The remainder of the total discharge is available for storage; how many acre feet is this?

The following table shows the percentage of the annual discharge that the river furnishes each month and the percentage of the total annual discharge actually needed for irrigation during each month of the dry season.

	Jan.	Feb.	Mar.	Apr.	May.	June.
Natural supply	4.1%	4.7%	5.2%	10.8%	5.3%	23.5%
Amount needed....	—	—	—	—	—	26.8%
	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Natural supply.....	18.0%	8.3%	5.8%	4.7%	4.5%	5.1%
Amount needed	29.7%	24.8%	18.7%	—	—	—

3. Find from the data of Exercise 1 and the table the number of acre feet available from the Logan River without storage for each month of the year.

4. During which four months is the natural flow insufficient? What per cent more is needed in July than is furnished by the natural flow of the river? How many acre feet is this? This part is furnished by storage.

5. Find how many acre feet must be furnished by storage for each of the other 3 months of the dry season.

6. What is the total number of acre feet that must be stored during the rest of the year to supply the deficiency during the dry season?

Oral.

1. .05 of a number is what per cent of it?

2. 50% of a number is what part of it? 25%? 33⅓%?

3. How may we find what per cent one number is of another?

4. 12 is —% of 18.

5. 3 is —% of 9.

6. — is 4% of 20.

7. — is 10% of 80.

8. — is 33⅓% of 30.

9. 16 is —% of 48.

10. 8 is —% of 40.

11. 16 is —% of 80.

12. 25 is 200% of —.

13. 75 is 75% of —.

14. 20 is —% of 30.

15. 17 is —% of 34.

16. 100 lb. is what per cent of a ton? 1,800 lb.? 500 lb.?

17. 8 oz. is what per cent of a pound? 4 oz.? 1 oz.?

18. 18 in. is what per cent of a yd.? 9 in.? 27 in.?

Written.

19. The United States silver dollar contains 371¼ grains of pure silver; this is 90% of its weight; what is the weight of a silver dollar?

20. Bees wintered indoors require about $\frac{8}{15}$ as much food as when wintered out of doors. What percentage of the food required out of doors is saved by wintering indoors? If a colony requires 24 lb. of food when wintered indoors, how many pounds would it require when wintered out of doors?

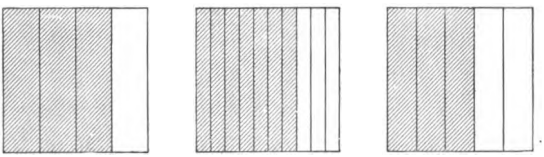
21. A man's tax was $150 at 1½%; the assessment was $\frac{2}{3}$ of the actual value of the property; what was its value?

22. According to the following table find the percentage of increase in the total number of votes cast for President of the United States in 1892 over that in 1888 within 1%.

1888.....	11,380,860	1896.....	13,923,102
1892.....	12,059,351	1900.....	13,959,653

23. As in the last exercise, find the percentage of increase in 1900 over 1896 to $\frac{1}{10}$ of 1%.

The squares represent the total population in 1900 of the states named, and the shaded areas represent the proportion of this population living in cities, (*urban* population).



Oral.

1. What per cent of the population of Massachusetts lived in cities? In the country?

2. Answer the same question for the other states.

Written.

3. The urban population of Connecticut was 55% of its total population; represent graphically as above.

4. Represent graphically the fact that 40% of the total population of the United States lived in cities.

5. The table shows relative prices from 1890 to 1903:

SUMMARY OF RELATIVE PRICES OF COMMODITIES, 1890 TO 1903, BY GROUPS

[Average price for 1890–1899 is 100.]

YEAR.	Farm products.	Food.	Clothing.	Fuel and lighting.	House furnishing goods.	Lumber, building materials.	Metals and implements.	Drugs and chemicals.	Miscellaneous.
1890	110.0	112.4	113.5	104.7	111.1	111.8	119.2	110.2	110.3
1891	121.5	115.7	111.3	102.7	110.2	108.4	111.7	103.6	109.4
1892	111.7	103.6	109.0	101.1	106.5	102.8	106.0	102.9	106.2
1893	107.9	110.2	107.2	100.0	104.9	101.9	100.7	100.5	105.9
1894	95.9	99.8	96.1	92.4	100.1	96.3	90.7	89.8	99.8
1895	93.3	94.6	92.7	98.1	96.5	94.1	92.0	87.9	94.5
1896	78.3	83.8	91.3	104.3	94.0	93.4	93.7	92.6	91.4
1897	85.2	87.7	91.1	96.4	89.8	90.4	86.6	94.4	92.1
1898	96.1	94.4	93.4	95.4	92.0	95.8	86.4	106.6	92.4
1899	100.0	98.3	96.7	105.0	95.1	105.8	114.7	111.3	97.7
1900	109.5	104.2	106.8	120.9	106.1	115.7	120.5	115.7	109.8
1901	116.9	105.9	101.0	119.5	110.9	116.7	111.9	115.2	107.4
1902	130.5	111.3	102.0	134.3	112.2	118.8	117.2	114.2	114.1
1903	118.8	107.1	106.6	149.3	113.0	121.4	117.6	112.6	113.6

Make a diagram similar to that on page 67 representing the prices in the first column, omitting the decimals; do the same for the second column.

6. Using this table, make and solve 10 problems similar to the problems of page 67.

IX

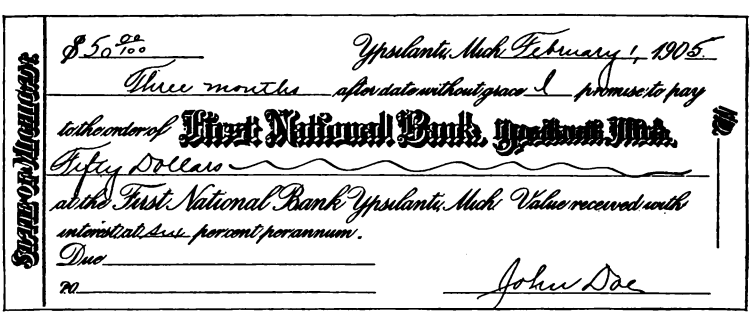
INTEREST AND BANKING

NOTES

Promissory Notes. When money is borrowed, the evidence of indebtedness is usually a written acknowledgment of the indebtedness and a promise to pay a certain sum of money at a certain time and place. Such a paper is called a *promissory note*, or simply a *note*.

Maker and Payee. The person who signs the note (agrees to pay) is the *maker* of the note; the person to whom it is to be paid is the *payee*.

Face and Amount. The sum loaned is called the *face* of the note, and the face plus the interest, the *amount*.


Promissory Note
$50 00/100
Ypsilanti, Mich. February 1, 1905.
Three months after date without grace I promise to pay
to the order of First National Bank, Ypsilanti, Mich.
Fifty Dollars
at the First National Bank Ypsilanti, Mich. Value received with
interest at six per cent per annum.
Due
20
John Doe

Who is the maker of the note above? The payee? What is its amount? Time? Rate of interest?

Write notes to cover the following conditions, all to be dated the day of writing:

	Maker.	Payee.	Amount.	Time.	Rate of interest per annum.
1.	John Smith	Henry Jones	$125	3 mo.	5%
2.	Richard Roe	John Doe	87	2 mo.	4%
3.	Yourself	Your teacher	580	6 mo.	6%

Maturity. A note is said to *mature* on the day when it is legally due. When the time is specified in days, exact days are counted in finding maturity; when in months, calendar months are counted. Generally, if a note matures on a holiday or on Sunday, it is payable the day following.

Days of Grace. Many states still allow **three days of grace** for the payment of notes. Thus, a 30-day note would be legally due in 33 days from date. When the law allows but does not compel their use, many notes are drawn to read "without grace."

Negotiable Notes. If the payee's name only is written in the note it can not be sold; but if it reads "*pay to the order of* ——," or "*to bearer*," it can be sold. Notes which may be sold are called *negotiable.*

Indorsement. When a note is transferred, the payee named must first write his signature across the back. This is called *indorsing* the note. Every indorser of a note is responsible for its payment, unless he writes the words "*without recourse*" before his name.

The responsibility of the indorser ceases upon the maturity of the note unless promptly notified of its non-payment, but the maker of the note is responsible for its payment until paid.

If the payee simply writes his name across the back, he is said to indorse it in blank. This makes it payable to the bearer. But if he designates to whom the note is to be paid he is said to indorse it in full.

Oral.

1. What is the face of the note shown on page 74?

2. When does the note mature without grace? With grace? Are days of grace allowed in your state?

Written.

3. Write an indorsement in full for a note to your order.

4. Write an indorsement for a note to your order so as not to become responsible for the payment of the note.

1. What is a usual rate of interest in your vicinity?

Written.

Use this rate to find the amounts of the following notes, treating 1 *mo. as* $\frac{1}{12}$ *of a year and* 1 *day as* $\frac{1}{30}$ *of a month:*

2. Face $175, date Jan. 10, time 1 yr. 6 mo.

3. Face $360, date Apr. 15, time 2 yr. 3 mo. 15 da.

4. Face $680, date Feb. 10, time 1 yr. 6 mo. 21 da.

5. Mr. Brown bought a house for $1,500. He paid $700 in cash and gave his note for the rest with interest at 5%, payable semiannually; what was the amount of interest due at each payment?

6. Mr. Lee bought a threshing machine for $300. He paid $150 in cash and gave his note for the rest with interest at 6%, payable annually; what was the amount of interest due at each payment?

7. A dealer sold an automobile for $1,000, receiving $400 cash and a note for the rest due in 3 years with interest at 6%, payable semiannually; how much interest was paid on the note altogether?

8. A farmer borrowed $600 on two equal notes, one due in 2 yr. and the other in 4 yr. with interest at 5%, payable annually; how much was due at the end of each year, if all payments were made when due?

9. Mr. Perkins bought a house and lot for $6,000. He paid $900 cash and gave three notes of equal amounts for the balance, due respectively in 4, 8, and 12 months with interest at $4\frac{1}{2}\%$; what sum was due at each payment?

10. Mr. Williams purchased an apartment building for $12,000. He paid $4,000 cash and gave five notes of equal amounts for the rest, payable respectively 6, 12, 18, 24, and 30 mo. after date, with interest at 5%, payable semiannually; what was each semiannual payment?

Privilege of Prepayment. Notes often contain a provision permitting the borrower to prepay a part of the principal before the note matures. Such advance payments are usually made on the days on which interest is due, or "interest days."

Oral.

1. A note for $500 dated Jan. 2, 1905, due in 5 yr. with interest at 6% payable annually provides the privilege of prepayment in multiples of $50 on interest days. When is the first interest due? How much is it? If the maker pays $100 of the face besides the interest, how much does he still owe?

2. What is the interest Jan. 2, 1907? If $100 of the face is paid on this date, how much does the maker still owe?

3. If the maker pays $100 on the principal on each interest day, what is the last payment?

4. If the payee of the above note pays only the interest on interest dates, what amount is due at the date of maturity of the note?

5. A promissory note for $2,500 due in 4 yr. with interest at 4% payable annually includes the privilege of prepayment in multiples of $500 on interest days. If the payee prepays $500 each interest day, what is the last payment?

Written.

6. On July 1, 1904, John Doe signed a note for $400 due in 3 yr., interest 5%, payable semiannually with the privilege of prepayment in multiples of $50 on interest days. He paid $150 and interest on the obligation at each opportunity. What was the interest Jan. 1, 1905? What was the amount still due?

7. Find the amount still due after each further payment in Exercise 6 until the obligation was settled.

Indorsement of Payments. In partial payment notes it is customary to indorse the payments on the back of the note, when made.

Written.

1. A note for $500, dated Jan. 2, 1905, with interest at 6%, payable annually for 3 yr. with a privilege of prepaying in multiples of $100 on interest days, had the following indorsements:

Received, Jan. 2, 1906, $130. John Doe.
" " 2, 1907, 224. John Doe.

Each indorsement includes the interest paid on that date; how much was due Jan. 2, 1908?

2. Upon a note for $350 dated July 1, 1903, with interest at 5%, payable semiannually, the following payments, including interest, were indorsed:

Received, Jan. 1, 1904, $58.75. Richard Roe.
" July 1, 1904, 82.50. Richard Roe.
" Jan. 1, 1905, 105.63. Richard Roe.

What was due July 1, 1905?

In the following notes the interest is payable annually, and prepayment of any sum is permitted on interest dates. Find the amount due at the date of settlement:

	Face.	Rate.	Date of Note.	Indorsements.	Date of Settlement.
3.	$525	4%	June 15, 1901	June 15, 1902, $125 June 15, 1903, 250 June 15, 1904, 125	June 15, 1905
4.	$250	6%	April 1, 1902	April 1, 1903, $120 April 1, 1904, 110	April 1, 1905
5.	$1,000	5½%	Aug. 20, 1901	Aug. 20, 1902, $425 Aug. 20, 1903, 425	Aug. 20, 1904
6.	$500	6%	May 1, 1900	May 1, 1901, $180 May 1, 1902, 121 May 1, 1903, 115 May 1, 1904, 109	May 1, 1905

Written.

1. Mr. Peters buys a house for $6,500, on which he pays $500 cash, and the remainder in monthly payments of $25, with interest at 6%.

That is, at the end of each month he pays the interest for one month on the sum still due, and besides pays an *instalment* of $25 on that sum. For example, at the end of the first month he pays $25 and the interest on the $6,000 for one month, leaving $5,975 still due. How much is his first monthly payment? The second? The third?

How much has the sum due been reduced in one year? What is still due at the end of the first year?

2. What was his payment at the end of the first month of the second year? The second month?

3. What was his payment at the end of the first month of the third year? The second month? How much has he paid off in ten years? What was his payment at the end of the first month of the eleventh year? The second month?

4. How long would it take him to pay for the house completely? What was his last payment?

FOR SALE—A BARGAIN

$7,450

Only $750 cash and $50 per month; 4912 Washington Place; interest 4 1-2 per cent; 2-story stone front 8-room residence; quarter-sawed oak finish, steam heat; tile bathroom. Open for inspection.

5. Make and solve six problems about the above advertisement.

6. Miss Martin bought a piano for $400 on the instalment plan, paying $40 at the time of purchase, and the remainder in monthly instalments of $10 each, plus the interest on the unpaid balance at 6% per annum; what was her first monthly payment? Her second? Her third?

7. What was her payment at the end of the first month of the second year? How long did it take to pay for the piano?

Exact Interest. Heretofore, in computing interest we have regarded a month as $\frac{1}{12}$ of a year and 1 day as $\frac{1}{30}$ of a month. But the government and some banks use 365 days as a year and the exact number of days between dates. Interest calculated in this way is called *exact interest.*

Thus, the exact interest on \$150 for 55 da. is the interest for $\frac{55}{365}$ of a year, or $\frac{\overset{1}{\cancel{55}}\times .06\times \$150}{\underset{73}{\cancel{365}}} = \frac{\$99}{73} = \$1.36.$

The following calendar shows the exact number of days between any day of any month and the same day of any other month within a year, leap years excluded:

Thus, from February 17 to July 17 is 150 da. The 150 is found opposite February and under July. Also, from February 17 to July 29 is 150 da. (found from the table) + 12 da. or 162 da. Similarly for all other cases.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January...	365	31	59	90	120	151	181	212	243	273	304	334
February..	334	365	28	59	89	120	150	181	112	242	273	303
March.....	306	337	365	31	61	92	122	153	184	214	245	275
April.....	275	306	334	365	30	61	91	122	153	183	214	244
May.....	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	335	365	30	61	92	122	153	183
July.....	184	215	243	274	304	335	365	31	62	92	123	153
August....	153	184	212	243	273	304	334	365	31	61	92	122
September.	122	153	181	212	243	273	303	334	365	30	61	91
October....	92	123	151	182	212	243	273	304	335	365	31	61
November.	61	92	120	151	181	212	242	273	304	334	365	30
December .	31	62	90	121	151	182	212	243	274	304	335	365

Written.

1. Find the exact interest on \$6,000 from May 5 to August 16 at 6%.

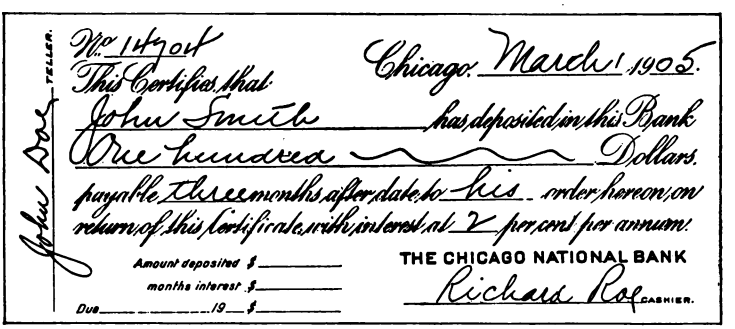
SUGGESTION.—Indicate the interest on \$6,000 for 1 yr. at 6%. For 1 da. For the exact number of days. Finally cancel and reduce.

Find the exact interest on:

2. \$800 from April 15 to June 21 at 5%.
3. \$15,000 from May 29 to June 24 at 4%.
4. \$3,650 from April 2 to May 31 at 5%.
5. Make and solve 10 other problems, using the calendar.

Interest on Deposits. No interest is paid by banks on money subject to check, unless the daily balance is regularly quite large. Banks will usually pay interest on money not subject to check. In this case a **certificate of deposit** is issued, stating conditions of deposit. If the money is repayable at a fixed time named in the certificate it is a **time certificate of deposit.** If the money is repayable at any time the holder of the certificate may desire, it is a **demand certificate of deposit.**

Usually no interest is paid on a demand certificate unless the money is allowed to remain at least a specified time.



Written.

1. Is the above a time or a demand certificate of deposit? Find the numbers to fill the blanks in the lower left corner of the certificate.

The following demand certificates of deposit bear interest at 2% per annum provided the deposit is allowed to remain at least 60 da.; find the amount of interest received in each case:

	FACE.	DATE OF DEPOSIT.	DATE OF PAYMENT.
2.	$125	Sept. 5, 1904	July 5, 1905
3.	50	Sept. 16, 1904	Oct. 16, 1905
4.	200	Jan. 2, 1905	May 2, 1905
5.	175	July 5, 1905	Sept. 16, 1905

Face. The amount for which the certificate is drawn is called its *face.*

If the deposit is made for a definite time, the interest is sometimes computed in advance, and included in the face of the certificate. The certificate then bears no further interest.

Written.

1. Mr. Walker deposited $150 in the Second National Bank of St. Louis for 4 months, with interest at the rate of $2\frac{1}{2}\%$ per annum. What was the face of his certificate, drawn to include the interest?

Compute the faces of the following certificates of deposit, including interest:

	Amount Deposited.	*Time.*	*Rate of Interest.*	*Face.*
2.	$225	6 mo.	3%	—
3.	120	8	$3\frac{1}{4}$	—
4.	450	3	$2\frac{1}{2}$	—
5.	670	2	$2\frac{1}{4}$	—
6.	1,500	2	$2\frac{1}{4}$	—
7.	2,500	3	$2\frac{1}{2}$	—
8.	2,750	10	4	—

9. Write a demand certificate of deposit with face $225, bearing 2% interest, and payable to yourself.

Oral.

10. What is the difference between a time certificate and a demand certificate?

11. When certificates are cashed a few days after date, what is the usual rule about paying interest?

12. A certificate of deposit for $500, dated June 2, 1903, bore 2% interest, limited to 1 yr.; when should a new certificate have been taken out in order to lose no interest? What would have been its face including the interest on the old certificate?

Many banks receive savings deposits in sums from $1.00 upward and pay interest thereon. The usual conditions are :

a. The entry in the depositor's book serves as a certificate of deposit.

b. Interest is computed semiannually; January 1st and July 1st. Usual rates of interest are 3%, 3½% and 4%. In the problems below use 3%, or the rate customary in your vicinity.

c. Interest is not computed for fractions of a month, or on fractional parts of a dollar. Many banks do not compute interest for fractions of a quarter (3 months). In some banks deposits made during the early days of the month (or quarter) are credited with interest from the beginning of the month (or quarter). The number of days thus allowed varies from five to fifteen.

d. The interest when computed is entered in the depositor's book, and draws interest thereafter.

e. The bank may require 60 days' written notice of any desired withdrawal of money, but usually pays on demand.

The conditions governing savings deposits vary somewhat in different parts of the country. If convenient, the conditions in vogue in banks known to the pupils may be ascertained and used in solving the problems.

Written.

1. Mr. Brown deposits $50 on Jan. 5, $40 Feb. 3, $80 Apr. 7; find the amount of interest credited to him July 1, if the bank pays no interest for parts of a quarter, but allows 10 days' grace. How much has he then on deposit?

2. Mr. Hay deposited in the bank mentioned in Exercise 1: July 6, $80; Sept. 23, $125; Oct. 8, $200; Nov. 1, $150. How much interest was credited Jan. 1?

3. A man deposited $15 in a savings bank regularly the first of each month, beginning July 1st, 1904. The bank computed interest from the date of deposit. How much interest was credited Jan. 1, 1905? If he continued his deposits, how much interest was credited July 1, 1905? How much had he then on deposit altogether?

87 DOLLARS.

$87

3%	4%	5%	6%	TIME.	8%	7%	10%	12%
				DAYS' GRACE.				
.24	.32	.40	.48	33	.64	.56	.80	.96
.46	.61	.76	.91	63	1.22	1.07	1.52	1.83
.67	.90	1.12	1.35	93	1.80	1.57	2.25	2.70
				DAYS.				
.01	.01	.01	.01	1	.02	.02	.02	.03
.01	.02	.02	.03	2	.04	.03	.05	.06
.02	.03	.04	.04	3	.06	.05	.07	.09
.03	.04	.05	.06	4	.08	.07	.10	.12
.04	.05	.06	.07	5	.10	.08	.12	.15
.04	.06	.07	.09	6	.12	.10	.15	.17
.05	.07	.08	.10	7	.14	.12	.17	.20
.06	.08	.10	.12	8	.15	.14	.19	.23
.07	.09	.11	.13	9	.17	.15	.22	.26
.07	.10	.12	.15	10	.19	.17	.24	.29
.08	.11	.13	.16	11	.21	.19	.27	.32
.09	.12	.15	.17	12	.23	.20	.29	.35
.09	.13	.16	.19	13	.25	.22	.31	.38
.10	.14	.17	.20	14	.27	.24	.34	.41
.11	.15	.18	.22	15	.29	.25	.36	.44
.12	.15	.19	.23	16	.31	.27	.39	.46
.12	.16	.21	.25	17	.33	.29	.41	.49
.13	.17	.22	.26	18	.35	.30	.44	.52
.14	.18	.23	.28	19	.37	.32	.46	.55
.15	.19	.24	.29	20	.39	.34	.48	.58
.15	.20	.25	.30	21	.41	.36	.51	.61
.16	.21	.27	.32	22	.43	.37	.53	.64
.17	.22	.28	.33	23	.44	.39	.56	.67
.17	.23	.29	.35	24	.46	.41	.58	.70
.18	.24	.30	.36	25	.48	.42	.60	.73
.19	.25	.31	.38	26	.50	.44	.63	.75
.20	.26	.33	.39	27	.52	.46	.65	.78
.20	.27	.34	.41	28	.54	.47	.68	.81
.21	.28	.35	.42	29	.56	.49	.70	.84
.22	.29	.36	44	30	.58	.51	.73	.87
				MONTHS.				
.44	.58	.73	.87	2	1.16	1.02	1.45	1.74
.65	87	1.09	1.31	3	1.74	1.52	2.18	2.61
.87	1.16	1.45	1.74	4	2.32	2.03	2.90	3.48
1.09	1.45	1.81	2.18	5	2.90	2.54	3.63	4.35
1.31	1.74	2.18	2.61	6	3.48	3.05	4.35	5.22
1.52	2.03	2.54	3.05	7	4.06	3.55	5.08	6.09
1.74	2.32	2.90	3.48	8	4.64	4.06	5.80	6.96
1.96	2.61	3.26	3.92	9	5.22	4.57	6.53	7.83
2.18	2.90	3.63	4.35	10	5.80	5.08	7.25	8.70
2.39	3.19	3.99	4.79	11	6.38	5.58	7.98	9.57
2.61	3.48	4.35	5.22	12	6.96	6.09	8.70	10.44
				YEARS				
5.22	6.96	8.70	10.44	2	13.92	12.18	17.40	20.88
7.83	10.44	13.05	15.66	3	20.88	18.27	26.10	31.32
10.44	13.92	17.40	20.88	4	27.84	24.36	34.80	41.76
13.05	17.40	21.75	26.10	5	34.80	30.45	43.50	52.20
3%	4%	5%	6%	TIME.	8%	7%	10%	12%

Interest Tables. Bankers and others who frequently compute interest use an *interest table.*

A part of an interest table is shown on the opposite page. The complete table consists of a similar page for each dollar from \$1 to \$99, for the multiples of \$100 from \$100 to \$900, and for the multiples of \$1,000 from \$1,000 to \$9,000.

Use of Table. The interest on \$87 at 3% for 33 da. is found in the column headed 3%, opposite to 33 in the central or time column. Result, \$.24. Similarly the interest on \$87 at 4% for 15 da. is \$.15, found in 4% column opposite to 15 in the time column.

Oral.

Find from the table the interest on \$87 for:

1. 26 da. at 5%.
2. 17 da. at 3%.
3. 7 mo. at 7%.
4. 27 da. at 12%.
5. 4 yr. at 4%.
6. 11 mo. at 8%.

7. A note for \$87 runs 90 da. at 6% in a state where days of grace are allowed. What is the interest?

8. What is the interest on \$87 for 2 yr. 3 mo. 18 da. at 6%?

SOLUTION.

1.	Interest for	2 yr.	(from table)..			\$10.44
2.	"	" 3 mo.	"	"	..	1.31
3.	"	" 18 da.	"	"	..	26
4.	"	" 2 yr. 3 mo. 18 da....				\$12.01

Written.

What is the interest on \$87 for:

9. 7 mo. 8 da. at 6%.
10. 5 mo. 13 da. at 7%.
11. 1 yr. 2 mo. 28 da. at 4%.
12. 3 yr. 6 da. at 5%.

The table can easily be used to compute interest at rates not directly given. Thus, to compute interest at $2\frac{1}{2}$%, take $\frac{1}{2}$ of the interest at 5% obtained from table. To obtain interest at $4\frac{1}{2}$%, take $\frac{3}{4}$ of the interest at 6% found from table.

What is the interest on \$87 for:

13. 5 mo. 10 da. at $2\frac{1}{2}$%.
14. 5 yr. at $3\frac{1}{2}$%.
15. 2 yr. 9 mo. at $4\frac{1}{2}$%.
16. 10 mo. 23 da. at 2%.

As we have only one page of the table the principal must remain \$87 in our problems. With the complete table interest on any principal is found similarly. To find the interest on \$7,849, the interest would be found (for the given time and rate) on the pages for \$7,000, \$800, \$49 and these amounts added.

Bank Discount. If a holder of a promissory note desires to use the money promised before the time of payment, a bank will advance him the money, provided he can give satisfactory guarantee that it will be paid when due. This is called *discounting* the note, and the compensation of the bank for this service is called *bank discount.*

Discount, like interest, is computed on the face of the note at a certain rate per cent per annum for the time from the day on which the note is discounted until it is due.

In some localities both days are counted, and the method of exact interest is used. In solving the problems below, use 360 da. as 1 yr., and do not include the day of discount.

Proceeds. The face of the note less the discount constitutes the sum paid to the holder and is called the *proceeds.*

Thus, if a note for $100 having 60 days to run is discounted at 6%, the discount is $.99, and the proceeds $99.01.

Written.

1. Find the discount and proceeds of a note for $3,650, without interest, dated Aug. 5, 1905, time 60 da., and discounted Aug. 29 at 6%.

PLAN. 1. Aug. 5 + 60 da. = Oct. 4. Aug. 29 to Oct. 4 = 36 da.
2. $\frac{36}{360}$ of .06 × $3,650 = $—, discount.
3. $3,650 − $21.90 = —, proceeds.

Find the proceeds:

	Date of Note.	Face.	Time.	Date of Discount.	Rate of Discount.
2.	June 3, 1905	$1,000	60 da.	June 25	6%
3.	July 15, 1905	3,500	90 da.	Aug. 1	5%
4.	May 23, 1905	8,000	60 da.	June 26	6%

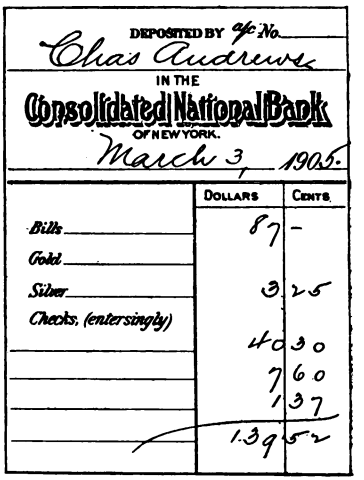
Discount on Interest-bearing Notes. If a note bears interest, the discount is usually computed upon the face plus the interest to the date of maturity. The method varies.

Thus, the discount on a note of $500, with interest at 6% for 3 mo., discounted 1 mo. after date, is found as follows:

1. $500 + $\frac{1}{4}$ × .06 × $500 = $507.50.
2. $\frac{1}{6}$ × .06 × $507.50 = $5.08, the discount.

5. If notes in Ex. 2, 3, 4 bear interest at 5%, find proceeds.

Deposit Slips. In offering money for deposit at a bank, it is customary to file a memorandum (*deposit slip*) similar to the one shown here.


DEPOSITED BY Chas Andrews No.
IN THE
Consolidated National Bank
OF NEW YORK.
March 3, 1905.

	DOLLARS	CENTS
Bills	87	-
Gold		
Silver	3	25
Checks, (enter singly)		
	40	30
	7	60
	1	37
	139	52

1. What was the total amount deposited according to the deposit slip shown?

Written.

2. What was the amount deposited by means of checks? by means of bills and silver?

3. Find the total amount of deposits made by a merchant on each of the following dates:

ITEMS.	Nov. 6.	Nov. 13.	Nov. 20.	Nov. 27.
Gold.....	$50	$50	$20	$10
Silver.....	230	189	300	40
Bills.....	185	264.50	160	300
Checks.....	380.59	740.82	590.94	1,265.17

4. A bank has a capital of $150,000 and a surplus (undivided profits) equal to 101.32% of this; find the surplus.

5. Find the surplus in each case as in Exercise 4:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
Capital..	$250,000	$500,000	$300,000	$750,000	$100,000
Surplus..	210.73%	134.82%	120.54%	119.73%	120.22%

BE YOUR OWN LANDLORD

6405 JACKSON AVE.

For Sale: New detached 2-story brick and stone residence, containing parlor, dining-room, kitchen, 4 bedrooms and bath; an ideal home. Price $5,500. ONLY $500 CASH and $40 monthly payments required to buy this house.

6. Make and solve six problems about the annexed advertisement, using $5\frac{1}{2}\%$ as the rate of interest.

Oral.

1. What is meant by a promissory note? By the maker of a note? The payee?

2. How would you word a promissory note for $200 dated to-day and payable in three years with interest at 5%?

3. What is meant by days of grace? Are days of grace legal in your state?

4. What is meant by negotiable? What is indorsing a note? Illustrate an indorsement in blank; in full.

5. How is exact interest computed? What do business men commonly use in computing interest?

6. What is a certificate of deposit? A demand certificate? A time certificate?

7. What do savings banks usually issue instead of certificates of deposit?

8. How is the interest computed on deposits made in a savings bank?

9. What is bank discount? Illustrate your answer.

10. What is meant by proceeds? Illustrate your answer.

11. How does a partial payment note differ from an ordinary promissory note? How is the interest computed on such a note?

12. What is the interest at 5% on a promissory note for $500 for 6 mo.?

13. What is the amount of the promissory note of Exercise 12?

14. When does a 6 mo. note mature, dated Jan. 2, 1905, if drawn in a state where grace is allowed?

15. How may one indorse a note and not become responsible for its payment?

16. What is the bank discount at 6% on a note for $200 due in 3 mo.?

Written.

1. Find the exact interest on \$6,000 from July 5 to Sept. 16 at 6%.

2. Find the discount and proceeds of the following note: Face, \$550; date of note, May 3, 1905; time to run, 6 mo.; date of discount, May 3, 1905; rate of discount, 5%.

3. Find the discount and proceeds of the following note: Face, \$3,500; time, 90 da.; discounted on the day of date at 6%.

4. Mr. Black sold a suburban farm of 12 acres. The land was sold at \$800 per acre, and the buildings at \$2,500. The purchaser paid \$3,000 cash and gave a note for the balance at $5\frac{1}{2}\%$ per annum. What was his annual interest payment?

5. Mr. Black lent the \$3,000 received at $4\frac{3}{4}\%$ per annum. What was Mr. Black's total annual income, according to Exercise 4, from the sale of his farm?

6. A merchant bought 4 boxes of tea containing 125 lb. each. The price was 76¢ per lb. cash, but the seller agreed to wait 3 mo., charging 6% interest. What was due at the end of 3 months?

7. A man bought a lot for \$480. He paid \$10 down and agreed to pay \$10 each month thereafter, together with interest at the rate of 6% per annum on the unpaid balance. What was his first monthly payment? His second payment? His fourth?

8. A demand certificate of deposit for \$800 dated May 3, interest at 2%, was presented for payment Sept. 29. How much interest was received?

9. Mr. George deposited \$50 in the savings bank Feb. 1; \$75 Mar. 1; \$100 Apr. 1 and \$50 May 1. How much interest, allowed from date of deposit, was credited to him July 1?

10. A 90-day note dated Jan. 5, face \$6,000, was discounted Mar. 1 at 6%. Find the proceeds.

X

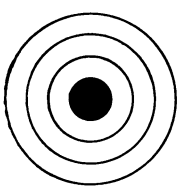
FORM STUDY AND MEASUREMENT

CIRCLES

Written.

1. Review the definitions and results of page 26. What is the cost of a floor for a round tower 40 ft. in diameter at 32¢ per square foot?

2. A round tree trunk has a circumference of 14.13 ft.; what is its diameter? What is the area of a cross section? Use $\pi = 3.14$.



3. A target is 8 ft. in diameter, the bull's-eye is 2 ft. in diameter, and each of the rings formed by the other circles is 1 ft. wide; what part of the whole target is the bull's-eye? Each ring?

4. The rotunda of the capital in Washington is a circular hall 98 ft. in diameter; find the cost of a stone floor for it at 49¢ per square foot. Use $\pi = \frac{22}{7}$.

5. A semicircular platform 20 ft. in diameter is to be carpeted with carpet 1 yd. wide, costing $1.20 a yard. Allowing 5% of the area of the platform for waste in fitting, what is the cost?

6. Find the cost of clearing a circular skating space 500 ft. in diameter, if the snow is 7 in. deep on the ice and the cost of removal is 15¢ a cubic yard.

7. A cow is tethered in the center of a field 100 ft. square with a rope 50 ft. long; over how many sq. ft. can she graze? Over what fractional part of the field?

8. The earth is about 8,000 mi. in diameter and makes one rotation on its axis in 24 hours; how far does a point on the equator move in a day? An hour? A minute?

Written.

1. Review the definitions and results of pp. 28–29. How many square feet of sheet-iron will it take to make 10 lengths of stovepipe 6 inches in diameter, each length being 18 in. long? (Use $\pi = \frac{22}{7}$.)

2. What will it cost to paint the surface of a standpipe 52 ft. in circumference and 68 ft. high, with open top, at 35¢ per square yard?

3. How many cubic yards of water will the pipe hold?

4. The pit for a turntable is 24 ft. in diameter and 4 ft. deep. Find the cost of excavation at 70¢ per cubic yard.

5. A sprinkling wagon has a cylindrical tank 12 ft. long and 4 ft. in diameter; how many gallons does it contain? (Use $7\frac{1}{2}$ gal. = 1 cu. ft.)

6. What will it cost to paint the sides and top of a gas tank 300 ft. in circumference and 60 ft. high at 33¢ per square yard?

7. How many cubic feet of gas will the tank contain?

8. A drain pipe has an internal diameter of 30 in. It is half full of water which flows at the rate of 2 miles per hour; how many cubic feet of water pass any point per minute?

9. A pillar 21 ft. high and 3 ft. in diameter is to be covered on the Fourth of July; equal quantities of red, white, and blue bunting are used. How many yards of each (one yard wide) must be bought?

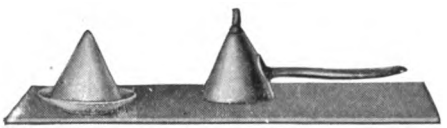
10. At 7 A.M. a gas tank 220 ft. in circumference is 10 ft. high. At 5 P.M. it has been raised to the height of 40 ft. by the manufactured gas, none having been consumed; how many cubic feet of gas have been manufactured per hour?

11. The production continuing at the same rate, the height of the tank had diminished to 4 ft. by 1 A.M. Find in cubic feet the average hourly consumption from 5 P.M. to 1 A.M.

Instruments: Rule, compasses, and scissors

1. Draw a circle of 3 in. radius. Cut along the circle and along a diameter.

2. Roll one of the pieces of paper into a form like the ice-cream mold and place it upright on a sheet of paper. Such a figure is called a **cone.**

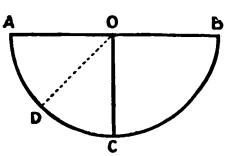


3. Mark around the base of the cone. What is the figure thus made?

4. Measure the diameter of this circle. Compute its circumference.

5. Compare the result with the length of the original circle. Why should this be so?

6. Draw on paper a semicircle of 4-inch radius. Fold through the middle so as to make a line like O C in the figure. Fold O A and O C so as to make the line O D. Cut along this line.



7. What part of the area of the circle is that of the sector, O D C B? What is the area of the sector?

8. Roll the sector into a cone; what is the area of the surface of this cone?

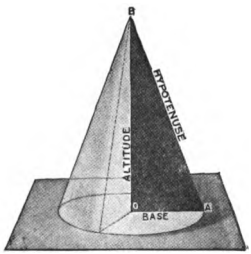
9. From a circle of 2-inch radius cut a sector whose area is $\frac{1}{4}$ that of the circle; how many sq. in. in the area of the sector? What would be the area of the cone thus made?

10. What is the area of the surface of a cone made from a sector that is $\frac{3}{4}$ of a circle of radius 6 in.?

11. Lulu rolled a cone on a paper, keeping its vertex stationary, and marked the path traced by the base until the cone made one turn. She connected the ends of the arc traced with the position of the vertex. What kind of a figure was formed?

1. Cut from cardboard a right-angled triangle having one perpendicular side longer than the other.

2. Set the triangle upright on a piece of paper, as shown in the picture, and rotate it about the vertical line O B.



3. What figure does the point A trace on the paper?

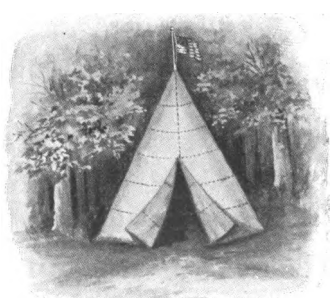
4. As the triangle rotates about B O, what kind of a figure does the hypotenuse generate?

Slant Height, Vertex, and Altitude of a Cone. The length of the hypotenuse is called the *slant height* of the cone. The top point of the cone is called its *vertex*. The perpendicular distance from the vertex to the base is called the *altitude*.

5. When a sector of a circle is made into a cone what does the arc of the sector become? What does the radius of the sector become? How is the area of a sector found?

The lateral area of a cone is $\frac{1}{2}$ the product of the perimeter of its base and the length of the slant height.

6. How many square inches of material are there in the crown of this hat, if the band is 18 in. long?



7. Roy and Harold made a tent for camping. The radius of the base was 6 ft. and the slant height 16 ft. How many square feet were there in its surface?

8. How many sq. yd. of duck were needed to make this tent? What did it cost at $12\frac{1}{2}¢$ a yd., allowing $\frac{1}{10}$ for waste?

1. Jennie wrapped a piece of paper about a cone (Fig. 1) and about a cylinder (Fig. 2) having a base and altitude equal to those of the cone; what was the shape of the piece of paper in each case?

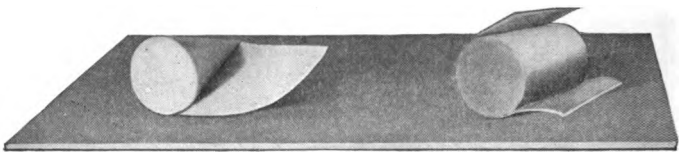


Fig. 1. Fig. 2.

She rolled the sector into a cone and the rectangle into a cylinder and found that three conefuls of sand just filled the cylinder.

2. If convenient, perform the same experiment.

3. The volume of the cylinder was how many times that of the cone? The volume of the cone was what part of that of the cylinder?

4. How did their bases and altitudes compare? How is the volume of a cylinder computed?

5. How may the volume of a cone be computed?

6. Sanford made from clay a cone and a cylinder having equal bases and altitudes. He weighed each and found that the cylinder weighed three times as much as the cone. What did this teach him about calculating the volume of a cone?

7. If convenient, repeat Sanford's experiment.

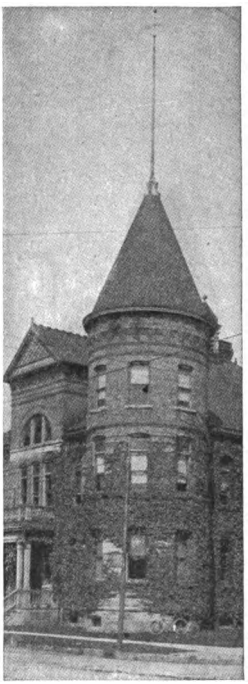
The volume of a cone is $\frac{1}{3}$ of the product of the altitude and the area of the base.

Find the numbers to fill the blanks:

	8.	9.	10.	11.
Altitude	3 in.	6 ft.	———	3 yd.
Base	12 sq. in.	40 sq. ft.	12 sq. ft.	———
Volume	———	———	144 cu. ft.	96 cu. yd.

Written.

1. The cylindrical tower in the picture is 40 ft. high to the roof. The outside diameter of the tower is 13 ft., and the thickness of the wall 6 in. Considering the tower as a complete cylinder, find the number of cubic feet in the wall, deducting $\frac{1}{8}$ of the volume of the wall for windows.



2. If 1 cubic foot of wall requires 23 bricks, how many bricks were used to build the tower?

3. Find the surface of the conical roof, its slant height being 20 ft. How many slates averaging 8 in. in width and laid 8 in. to the weather were used to cover the roof?

4. The diameter of a street roller is 3 ft. and its length 5 ft. How many square feet does it cover in 1 revolution?

5. How many revolutions are made in rolling a street 40 ft. wide in front of a 350-foot block?

6. On May 24, 1902, 1 in. of rain fell in Chicago in 1 hr.; how many cubic feet of water fell within the city? (area, 180 sq. mi.). How many cubic feet per minute? (Calculate all results to the nearest million.)

7. If the water that fell were stored in a cylindrical stand pipe 100 ft. in diameter, how high would the stand pipe have to be? Express the result to the nearest .1 mi.

8. What was the weight of the water. (1 cu. ft. weighs 62.5 lb.) Express the result to the nearest thousand tons.

XI

GENERAL REVIEW

PROBLEMS

Oral.

1. Add: 199; 198; 201; 202; 200; 197; 203.

2. Subtract: 199 from 763; 501 from 1,763; 298 from 763.

3. Multiply by 9: 17; 23; 12; 20; 25; 15.

4. What is the cost of 24 packages of cereal at 9¢ a package? Of 15 cakes of soap at 9¢ a cake? Of 19 yards of lace at 9¢ a yard?

5. Multiply by 99: 8; 9; 12; 15; 22; 25; 30.

6. Find the cost of 6 sacks of flour at 99¢ a sack; of 12 shirts at 99¢ each; of 7 dozen handkerchiefs at $1.99 a dozen.

7. What is the area of a floor 16 ft. by 25 ft.? What did it cost at 25¢ a square foot?

8. 100 oranges sold for $1.50; what was the rate per dozen?

9. 25 cans of corn sold for $3.75; what was the rate per dozen.

Find the cost of:

10. 20 lb. sugar at $5\frac{1}{4}$¢ a lb.

11. 8 lb. steak at 19¢ a lb.

12. 12 lb. coffee at $33\frac{1}{3}$¢ a lb.

13. 5 lb. 4 oz. tea at 80¢ a lb.

14. 12 lb. crackers at $12\frac{1}{2}$¢ a lb.

15. 2 oz. pepper at 35¢ per lb.

16. A grocer buys a 47-lb. bag of rice; how many 8-oz. packages can he make out of it?

Written.

Add:

1. $\frac{3}{4}$; $\frac{8}{9}$; $\frac{7}{8}$.
2. $\frac{1}{12}$; $\frac{9}{10}$; $\frac{7}{15}$.
3. $\frac{4}{5}$; $\frac{5}{7}$; $\frac{8}{15}$.
4. $2\frac{1}{2}$; $3\frac{2}{8}$; $5\frac{4}{5}$.
5. $12\frac{1}{2}$; $9\frac{3}{4}$; $1\frac{1}{8}$.
6. $6\frac{1}{3}$; $33\frac{2}{5}$; $2\frac{8}{9}$.

Subtract:

7. $\frac{9}{34} - \frac{3}{17}$.
8. $\frac{40}{9} - \frac{17}{18}$.
9. $\frac{23}{25} - \frac{4}{5}$.
10. $9\frac{1}{2} - 4\frac{7}{8}$.
11. $12\frac{3}{8} - 9\frac{7}{16}$.
12. $40\frac{3}{5} - 29\frac{3}{4}$.

Multiply:

13. $\frac{5}{8} \times \frac{10}{12}$.
14. $\frac{6}{7} \times \frac{13}{14}$.
15. $\frac{1}{2} \times \frac{3}{4} \times \frac{8}{9}$.
16. $1\frac{1}{2} \times 4\frac{3}{4}$.
17. $5\frac{3}{5} \times 6\frac{2}{3}$.
18. $12\frac{1}{2} \times 33\frac{1}{3}$.

Divide:

19. $\frac{3}{4} \div \frac{7}{16}$.
20. $\frac{4}{5} \div \frac{9}{10}$.
21. $\frac{3}{11} \div \frac{9}{22}$.
22. $4\frac{1}{2} \div 3\frac{3}{4}$.
23. $5\frac{1}{6} \div 1\frac{11}{12}$.
24. $8\frac{9}{10} \div 4\frac{3}{5}$.

25. Mr. Marsh's grocery bill for November contained items costing the following amounts:

Nov. *1-2*	*4-8*	*9-14*	*15-20*	*21-25*	*26-30*
$0.15	$0.53	$0.15	$0.10	$0.48	$0.18
.05	.75	.38	.75	.56	.30
.15	.25	.22	.30	.12	.12
.44	.22	.15	.26	.15	.20
.15	.10	.10	.15	.08	.28
.08	.35	.14	.13	.10	.20
.15	.10	.25	.18	.20	.10
.24	.10	.30	.24	.05	.10
.30	.15	.24	.06	.10	.09
.25	.22	.35	.09	.35	.15

Find the total of the bill.

26. What is the cost of a quarter of a ton of range coal at $7.25 a ton, and 100 lb. of cannel coal at $8.00 a ton?

27. A butcher sold a lamb as follows: leg, 22.2 lb. at 14¢; loin, 17.5 lb. at 11¢; rib, 14.5 lb. at 10¢; chuck, 19.8 lb. at 4¢. What did he receive altogether?

28. In Exercise 27 what was the average price per lb.? (To the nearest $\frac{1}{10}$¢.)

Written.

1. How many quarts in 4 bu. 3 pk. 7 qt.? In 10 bu. 1 pk. 5 qt.?

2. How many square inches in 1 sq. yd.? 5 sq. ft.? 100 sq. ft.?

3. How many yards, feet and inches in 1,620 in.? In 2,000 in.?

4. How many days, hours and minutes in 8,256 min.? In 12,000 min.?

Express decimally to the nearest thousandth:

5. $\frac{5}{8}$ ft. 6. $\frac{7}{12}$ yd. 7. $\frac{11}{64}$ gal. 8. $\frac{283}{200}$ mi.

9. $\frac{14}{125}$ bu. 10. $\frac{9}{32}$ hr. 11. $1\frac{4}{9}$ da. 12. $9\frac{13}{16}$ cu. yd.

13. Express decimally as a number of days: 3 da. 19 hr. 40 min.

14. Express decimally as a number of miles: 5 mi. 80 yd. 2 ft.

15. Express 7.65 gal. as a number of gallons, quarts and pints.

Add:

16.	17.	18.
17 da. 15 hr.	125 mi. 205 ft.	40 bu. 3 pk.
183 20	368 735	87 2

Subtract:

19.	20.	21.
53 gal. 1 qt.	12 yd. 2 ft.	127 cu. yd. 8 cu. ft.
19 3	7 2	99 19

Multiply:

22.	23.	24.
17 A. 40 sq. rd.	45 gr. 11 doz.	6 mi. 5,000 ft.
5	15	9

Divide:

25.	26.	27.
9)23 hr. 45 min.	15)226 yd. 1 ft.	121)400 ft. 9 in.

28. A chimney 115 yd. high is built at the rate of 2 yd. 1 ft. 8 in. per day; how many days are required to build it?

Written.

1. The area of Douglas park in Chicago is 179.79 acres. It is a rectangle about $\frac{1}{3}$ mi. wide; how long is it in miles? In feet?

2. A man set out geraniums 8 in. apart along a bed 5 yd. 2 ft. 4 in. long. If the end plants stand 4 in. from the edges of the bed how many plants were required?

3. What is the cost of a hardwood floor for a room 15 ft. by 18 ft. at 90¢ a square yard?

4. A rug 9 ft. wide covers 12 sq. yd. of floor; how long is the rug?

5. A manufacturer wishes to make 25 dozen tables with square ends, 44 in. wide and 10 ft. long; how many square feet of lumber are required to make the tops?

6. In 1900 there were manufactured in the United States 1,825,769 watch movements. If these were placed side by side on a straight line, find the length of the line, estimating that, on an average, one watch would require 2 in. (First make a rough estimate in miles.)

7. The fence around a rectangular corner lot is 279 ft. long. Find the frontage of the lot on both streets together.

8. A boy measured the side of the house with a stick which he thought was a yard long and reported the house to be 42 ft. long. He afterward found that his stick lacked 2 in. of being a yard long. What was the length of the house?

9. A room 21 ft. by 18 ft. is to be covered with carpet 27 in. wide, the breadths running the long way of the room. The cost of sewing is 3¢ per yard of seam; the cost of the carpet is \$1.12$\frac{1}{2}$ a yard, and the cost of laying, $\frac{1}{2}$¢ a yard. Find the cost of the carpet put down.

10. A farmer had 671 bu. of apples and put them into barrels holding 2 bu. 3 pk. each; how many barrels were required?

Written.

1. 4 is what per cent of 32? Of 24? Of 12? Of 100?

2. 5 is 10% of what number? 6 is 6% of what number?

3. A 10% cut in the wages of the Rhode Island cotton-mill operators reduced their weekly earnings $20,000; how much had they been receiving a week?

4. The wages of 75,000 New England operators earning $10 a week were cut 8%; what was the reduction of the total earnings of these operators a week?

5. Ice-cold water increases 4% of its volume, if heated to the boiling point; what is the volume of boiling water which measured 5 gal. 3 qt. 1 pt. when ice-cold?

6. If oak wood is burned the ashes weigh about 3% of the original weight of the wood; how many pounds of wood are burned in producing 5 lb. of ashes?

7. In a recent year 39,673 applications were made for patents at the Patent Office in Washington; 26,418 were granted; what per cent of the number of applications was this?

8. 1,802 of these applications were made by foreigners; what per cent of the total were made by Americans?

9. A large ocean passenger steamer requires provisions for one voyage about as follows:

20 tons of flour and 125% as many tons of potatoes.
2,000 qt. of milk and cream, and 75% as much ice cream.
35,000 lb. fresh meat; 8⅓% as much fish, and 14% as much salt meat.

How many pounds of fish are required? Of salt meat? How many quarts of ice cream? How many tons of potatoes?

10. The total coal production of the United States in 1903 was 359,421,311 tons, an increase of 19% over 1902; how many tons were produced in 1902?

11. The value of the coal produced during a recent year was $504,190,733, an increase of 38% over the previous year. What was the value of the product of the previous year?

Written.

Find the interest on:

1. \$75 at 6% for 3 yr. 4 mo.

2. \$50.25 at 4% for 2 yr. 8 mo. 15 da.

3. \$250 at $4\frac{1}{2}\%$ for 11 mo.

4. \$325 at 5% for 1 yr. 7 mo. 14 da.

5. A merchant gained 8% on some goods which sold for \$5.40; how much did the goods cost?

6. What is a man's tax whose house and lot, valued at \$4,000, are taxed on $\frac{3}{4}$ of their value at $\frac{3}{4}\%$?

7. What does it cost to insure the same house for \$2,500 at 90¢ per \$100 annually?

8. A publisher offered a discount of 10% to schools and a further discount of 5% from the amount of the bill for cash; what was the cash cost of books for a school library whose published prices amounted to \$152?

9. A commission merchant sells 200 crates of cabbage at 30¢ a crate and charges 5¢ a crate for selling them; how much does he remit to the shipper?

10. A commission merchant sells 25 tubs of butter, each containing 56 lb. for 20¢ a pound, and charges 60¢ a tub for selling; what is his commission? How much does he remit to the shipper?

11. A lot having a frontage of 46 ft. on Adams St., Chicago, and a depth of 190 ft., is leased for 198 years at an annual rental of \$18,000. Estimating the annual rental to be 4% of the value of the ground, what is its value? What is the value per square foot?

12. In a lease for 198 years, the Marquette Safety Deposit Company agrees to pay \$10,500 annual rental for a piece of ground 26 ft. by 100 ft. What is the value of the ground, based on a 4% rental? The value per square foot?

XII

FUNDAMENTAL OPERATIONS

NOTATION AND NUMERATION

Systems of Notation. Two systems have been explained:

The **Arabic** or **Hindu** system of notation, with the symbols:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,

and the **Roman** system, with the symbols:

I, II, III, IV, V, VI, VII, VIII, IX, X, for the numbers 1–10, and L = 50, C = 100, D = 500, M = 1,000.

For the reading and writing of numbers in the Hindu system to billions review pages 10–11.

Integers. A whole number is called an *integer.*

Thus, 1, 2, 3, 15, 100 are integers.

Fractions. One or more of the equal parts of a quantity is called a *fraction.*

Thus, $\frac{1}{11}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{1}{3}$, are fractions.

Mixed Numbers. A number consisting of an integer and a fraction is called a *mixed number.*

Thus, $1\frac{1}{2}$, $3\frac{4}{5}$, $7\frac{8}{11}$ are mixed numbers.

Abstract and Concrete Numbers. Numbers considered apart from particular things are called *abstract* numbers; numbers of things are called *concrete* numbers.

Thus, 2, 3, $\frac{5}{8}$, and $1\frac{1}{2}$ are abstract numbers, while 5 words, 2 apples, $3\frac{1}{2}$ ft., and $\$\frac{3}{4}$ are concrete numbers.

Denominate Numbers. Numbers used in weights and measures are commonly called *denominate numbers.*

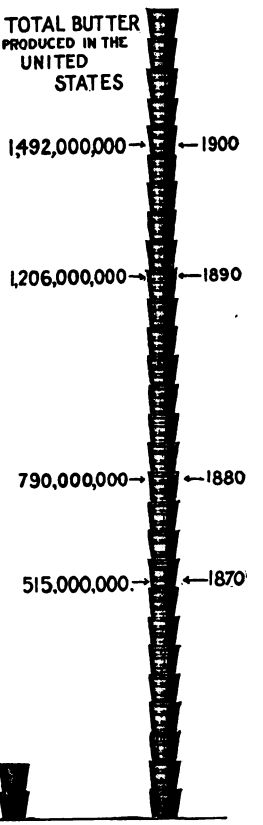
Thus, 8 lb., 12 gal., 3 qt., 2 bu., 2 pk., 6 qt., and $3.75 are denominate numbers.

NOTE.—Definitions should be understood, but not made subjects of class drill.

Oral.

1. Name the first three periods in the numeration of integers.

2. The diagram shows the number of million pounds of butter produced in the United States in 1870, 1880, 1890, and 1900. Read the amount produced in 1870; in 1880.



Written.

3. Write 200 millions in full with figures; 400 millions; 790 millions.

4. Write 1,000 millions with figures. How many periods are there in the number? What is the name of the fourth period? Read the number.

Periods. Numbers of more than four periods are very rarely needed. The three periods next in order are called **trillions, quadrillions, quintillions.**

5. How many pounds of butter were produced in 1890? In 1900? Write these numbers.

6. 100 is what power of 10? 1,000 is what power of 10?

Exponents. Powers of 10 larger than 1,000 are expressed more simply by the use of exponents.

Thus, 10,000 is 10^4; 50,000 is 5×10^4; 85,000 is 85×10^3.

7. 100,000 is what power of 10? Write it using an exponent; also 900,000; 950,000; 895,000; 1,000,000; 9,000,000; 129,000,000; 1,236,000,000.

8. Using exponents, write all of the numbers in the diagram expressing amounts of butter and oleomargarine.

Summary of Addition. In the five cases of addition shown at the right the principle is the same. Numbers of like denominations are arranged in columns and added separately. Each sum is expressed in units of the next larger denomination, whenever possible.

(1)
58
79
17 = 1 ten + 7 ones
12 tens
137

(2)
5.8
7.9
1.7 = 1 + 7 tenths
12 ones
13.7

(3)
$5\frac{8}{13}$
$7\frac{9}{13}$
$1\frac{4}{13}$ = 17 thirteenths
12 ones
$13\frac{4}{13}$

(4)
5 lb. 8 oz.
7 9
1 1 = 17 oz.
12 lb.
13 lb. 1 oz.

(5)
$5a + 8b$
$7a + 9b$
$17b = 17b$'s
$12a = 12a$'s
$12a + 17b$

Oral.

1. 8 ones and 9 ones are — ones. In example (1) how many ones are there in the right column?

This is how many tens and how many ones?

2. How many tenths are there in the sum of the right column of example (2)?

This is how many ones and how many tenths?

3. How many thirteenths are there in the sum of the right column of example (3)?

This is how many ones and how many thirteenths?

4. Show that in each example the sum of the right column is 17 units of a certain kind.

5. If $a = 10$ and $b = 1$, what other example does (5) become?

6. What is the sum of the left column in each example? In what unit is each expressed?

7. How must fractions with different denominators be expressed before adding?

Since pupils should add rapidly at this stage of the work, it would be well to fix a time limit for finding the correct result.

In adding, look for figures whose sum is 10, as 9 and 1; 8 and 2; 7 and 3; etc.

Thus, in Exercise 6, right column, it is easier simply to think, "1, 11, 21."

Written.

Add and test:

1.	2.	3.	4.
8275	1403	1127	4083
3268	3399	8943	986
7457	2401	1794	77

5.	6.	7.	8.
1812	916	566	1204
806	894	324	2963
809	403	973	3690
813	787	437	1784
427	321	238	4693

9.	10.	11.	12.
12.7	40.30	90.80	3.125
14.3	17.25	12.60	0.625
40.8	16.85	30.17	3.315
90.6	40.36	45.96	8.055

13. A clerk made the following sales in one forenoon: \$3.75, \$16.50, \$1.67, \$.35, \$20.13, \$5.00; what was the total amount of his sales?

Add:

14. $\frac{1}{2} + \frac{1}{4} + \frac{3}{8}$. **15.** $\frac{5}{6} + \frac{7}{12} + \frac{1}{3}$. **16.** $\frac{1}{5} + \frac{2}{3} + \frac{5}{6}$.

17. $\frac{1}{5} + \frac{1}{25} + \frac{3}{50}$. **18.** $\frac{9}{10} + \frac{3}{20} + \frac{17}{40}$. **19.** $\frac{1}{8} + \frac{3}{4} + \frac{3}{16}$.

20. $\frac{5}{16} + \frac{5}{8} + \frac{3}{4}$. **21.** $\frac{7}{12} + \frac{5}{6} + \frac{7}{8}$. **22.** $\frac{1}{5} + \frac{2}{7} + \frac{3}{11}$.

Add:

23.	24.	25.
7 yd. 2 ft.	12 gal. 2 qt.	12 mi. 100 ft.
3 1	19 1	93 950

NOTE.—The teacher will add as many such exercises as circumstances require. Some or all of pages 12–14, 32–34, 42, may also be reviewed.

Oral.

1. If a and b represent two numbers, what represents their sum?

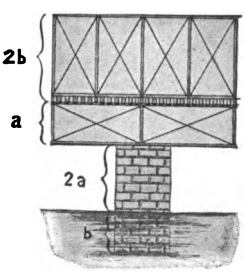
2. If $a = 3$ and $b = 2$, $a + b =$?

If a *and* b *stand for the following numbers, what numbers are represented by* a + b?

	3.	4.	5.	6.	7.	8.
a =	6	33	$\frac{3}{2}$	1.36	$1\frac{2}{3}$	1.4
b =	1	22	$\frac{1}{2}$	.08	$\frac{3}{5}$	.3

What are the sums of:

9. 4 yd. and 7 yd.?
10. $4a$ and $7a$?
11. $6a$, $4a$, $5a$?
12. $15x$ and $18x$?
13. $15b$, $2b$, b?
14. $12a$, $2a$, $3a$?
15. 12 lb., 2 lb., 3 lb.?
16. $5x$, $8x$?
17. $\frac{1}{2}x$, $\frac{1}{3}x$, $\frac{1}{4}x$?
18. $2n$, $7n$, $3n$, $11n$?



19. $5a + 7a + 2a + 3a$ are how many times a?

What may be written for each of the following:

20. 5 ft. + 2 ft. + 6 ft. + 4 ft.?
21. $a + a + a + a + a$?
22. $3c + 3c + 3c + 3c$?

23. (1) What expression represents the height of the top of this bridge above the top of the pier?

(2) What expression represents the height of the pier and its foundation.

(3) What is the height of the bridge from the bottom of the foundation?

Let $a = 8$ ft. and $b = 10$ ft. and find each of these heights. The work shows how the results of questions 1 and 2 may be added to answer question 3.

$$\begin{array}{r} a + 2b = 28 \\ 2a + \ b = 26 \\ \hline 3a + 3b = 54 \end{array}$$

Add:

24. 5 ft. 4 in.
    9 ft. 7 in.
25. $5a + 4b$
    $9a + 7b$
26. $5x + 4$
    $9x + 7$
27. $5m + 4r$
    $9m + 7r$

Summary of Subtraction. In the eleven examples of subtraction shown at the right the principle is the same. For convenience, numbers of like denominations are placed under each other.

In examples (6) to (9), it is necessary in subtracting to add to both minuend and subtrahend a unit of the order of the left column.

(1) 69 − 45 = 24

(2) 6.9 − 4.5 = 2.4

(3) $6\frac{9}{13} - 4\frac{5}{13} = 2\frac{4}{13}$

(4) 6 lb. 9 oz. − 4 lb. 5 oz. = 2 lb. 4 oz.

(5) $6a + 9b - (4a + 5b) = 2a + 4b$

(6) 43 − 27 = 16

(7) 4.3 − 2.7 = 1.6

(8) $4\frac{3}{13} - 2\frac{7}{13} = 1\frac{9}{13}$

(9) 4 lb. 3 oz. − 2 lb. 7 oz. = 1 lb. 12 oz.

(10) $7\frac{5}{12} - 3\frac{1}{3} = 4\frac{1}{12}$

(11) $9\frac{3}{10} - 6\frac{2}{3} = 2\frac{19}{30}$

Oral.

1. To how many units of the column on the right is a unit of the column on the left equal in Example (6)?

2. Answer the same question in Example (7); in Example (8); in Example (9).

3. Why are the answers similar in Examples (1) to (5), but not similar in Examples (6) to (9)?

4. What additional step enters in Examples (10) and (11)?

5. How must fractions be expressed before subtracting?

6. How is subtraction tested?

Written.

Subtract and test:

7. 7893 − 2469

8. 84.33 − 23.99

9. 8703 − 2604

10. 12.03 − 9.99

11. $8\frac{11}{12} - 3\frac{7}{12}$

12. 8 ft. 11 in. − 3 ft. 7 in.

13. $2\frac{5}{6} - 1\frac{1}{2}$

14. $6\frac{3}{4} - 5\frac{1}{8}$

15. $27\frac{1}{3} - 18\frac{2}{5}$

Written.

Subtract and test:

1. $\frac{7}{8} - \frac{3}{4}$.
2. $\frac{9}{10} - \frac{2}{5}$.
3. $\frac{6}{7} - \frac{1}{3}$.
4. $\frac{4}{5} - \frac{3}{25}$.

5. $8\frac{9}{10}$ − $3\frac{1}{2}$
6. $17\frac{5}{6}$ − $9\frac{7}{12}$
7. $10\frac{2}{3}$ − $8\frac{4}{5}$
8. $16\frac{1}{2}$ − $4\frac{8}{9}$

9. 12 gal. 1 qt. − 9 3
10. 3 pk. 2 qt. − 1 6
11. 40 bu. 1 pk. − 19 3

12. 6 mi. 100 ft. − 5 989
13. 80 lb. 6 oz. − 40 12
14. 40 gal. − 30 1 pt.

15. From the following table of facts about the United States postal service find the increase in the number of post-offices from 1865 to 1880; from 1870 to 1885; from 1890 to 1900; from 1865 to 1903.

Fiscal Year.	Number of Post-Offices.	Extent of Post Routes in Miles.	Revenue of the Department.	Expenditure of the Department.
1865	20,550	142,340	$14,556,159	$13,694,728
1870	28,492	231,232	19,772,221	23,998,837
1875	35,547	277,873	26,791,360	33,611,309
1880	42,989	343,888	33,315,479	36,542,804
1885	51,252	365,251	42,560,844	49,533,150
1890	62,401	427,991	60,882,097	65,930,717
1895	70,064	456,026	76,983,128	86,790,172
1896	70,360	463,313	82,499,208	90,626,296
1897	71,022	470,032	82,665,462	94,077,242
1898	73,570	480,462	89,012,618	98,033,523
1899	75,000	496,948	95,021,384	101,632,160
1900	76,688	500,982	102,354,579	107,740,268
1901	76,945	511,808	111,631,193	115,554,920
1902	76,215	507,540	121,848,047	124,785,697
1903	74,169	506,268	134,224,443	138,784,488

16. Make and solve five similar problems.

17. Make and solve ten similar problems about the extent of post routes; also make and solve ten about the amounts of revenue; also about the expenditures.

NOTE.—The teacher will add similar exercises if needed. Some or all of pages 14, 33, 34, 42, 44 may be reviewed.

Oral.

1. If a and b stand for two numbers, what stands for their difference? If $a = 5$, $b = 2$, then $a - b = ?$

Find the value of a − b *in the following cases:*

	2.	3.	4.	5.	6.	7.
$a =$	7	$3\frac{1}{2}$	8.9	16	$7\frac{1}{2}$	2.76
$b =$	5	$2\frac{1}{2}$	3.6	6.5	$3\frac{5}{6}$	1.95

8. 8 yd. − 5 yd. = — yd. **9.** $8a - 5a = -a$.

10. $\frac{5}{6}$ ft. − $\frac{1}{6}$ ft. = — ft. **11.** $\frac{5}{6}a - \frac{1}{6}a = -a$.

12. 3 mi. − 1.5 mi. = — mi. **13.** $3b - 1.5b = -b$.

14. 40 lb. − 25 lb. = — lb. **15.** $40c - 25c = -c$.

16. A grocer bought $5a$ bushels of apples and $4b$ bushels of pears; how many bushels of fruit did he buy?

17. He sold a bushels of apples and $2b$ bushels of pears; how many bushels of fruit did he sell?

$5a + 4b$ = the number of bushels of fruit the grocer bought.
$a + 2b$ = the number of bushels of fruit the grocer sold.
$4a + 2b$ = the number of bushels of fruit the grocer had left.

Let $a = 5$ and $b = 10$. How many bushels of fruit did the grocer buy? How many did he sell? How many had he left?

18. When $a = 1$ and $b = 1$, what is the value of $16a + 5b$? What is the difference of these values? Subtract $9a + 2b$ from $16a + 5b$ in the usual way. What is the value of the result for $a = 1$, $b = 1$? What mode of testing subtraction does this suggest?

Written.

Subtract and test:

19. $9b + 6c$ / $5b + 3c$

20. $\frac{5}{6}b + 1\frac{1}{2}d$ / $\frac{1}{2}b + \frac{7}{12}d$

21. $8.5a + 3.6b$ / $1.5a + 1.9b$

22. $86a + 7c$ / $19a + c$

23. $3a + 6b + c$ / $3a + \frac{1}{2}b$

24. $4m + n + 3p$ / $2m + p$

Summary of Multiplication. In the five cases of multiplication shown at the right, the principle is the same. Numbers of like denominations are multiplied separately. Each partial product is expressed in terms of the next larger denomination as far as possible. The partial products are arranged conveniently and added.

(1)
$$\begin{array}{r} 73 \\ \underline{\times\ 6} \\ 18 \\ \underline{42} \\ 438 \end{array}$$

(2)
$$\begin{array}{r} 7.3 \\ \underline{\times\ 6} \\ 1.8 \\ \underline{42} \\ 43.8 \end{array}$$

(3)
$$\begin{array}{r} 7\frac{3}{8} \\ \underline{\times\ 6} \\ \frac{18}{8}\ (2\frac{2}{8}) \\ \underline{42} \\ 44\frac{2}{8}\ (\frac{1}{4}) \end{array}$$

(4)
$$\begin{array}{r} 7\text{ ft. }3\text{ in.} \\ \underline{\times\ 6} \\ 18\text{ in. (1 ft. 6 in.)} \\ \underline{42\text{ ft.}\phantom{\text{ in.}}} \\ 43\text{ ft. }6\text{ in.} \end{array}$$

(5)
$$\begin{array}{r} 7a + 3b \\ \underline{\times\ 6} \\ 42a + 18b \end{array}$$

Oral.

1. Point out how the number 18 is obtained in each example; the number 42.

2. State the values that must be given to a and b in the last example to make it the same as each of the others, in turn.

3. Why does the number 18 occur in the final result of (5), but not in those of (1) to (4)?

Written.

Multiply and test by interchanging multiplier and multiplicand:

4. $\begin{array}{r} 467 \\ \underline{325} \end{array}$

5. $\begin{array}{r} 6794 \\ \underline{3005} \end{array}$

6. $\begin{array}{r} 999 \\ \underline{555} \end{array}$

7. $\begin{array}{r} 4603 \\ \underline{904} \end{array}$

8. $\begin{array}{r} 36.7 \\ \underline{.49} \end{array}$

9. $\begin{array}{r} 30.09 \\ \underline{9.7} \end{array}$

10. $\begin{array}{r} 4.771 \\ \underline{77} \end{array}$

11. Find $17 \times 18\frac{5}{8}$; $28 \times 16\frac{2}{3}$; $43 \times 11\frac{4}{5}$; $67 \times 19\frac{1}{2}$.

12. Find 8×9 lb. 9 oz.; 12×40 yd. 2 ft.

Oral.

1. How many zeros are annexed in multiplying an integer by 10? By 100? By 1,000?

2. Multiply by 10: 18, 180, 1,800, 63, 630, 876, 806.

3. Multiply by 100: 6, 60, 600, 13, 130, 106, 66, 660.

4. Multiply by 1,000: 3, 4, 5, 50, 23, 20, 145, 1,635.

5. How many places is the decimal point moved to the right in multiplying a decimal by 10? By 100? By 1,000?

6. Multiply by 10: .3, 9.4, 3.05, 16.67, 1.009, 90.09.

7. Multiply by 100: .06, .005, 6.05, 12.005, 12.635.

8. Multiply by 1,000: .008, 2.005, 2.63, 46.73, 965.5.

We have already seen that **a fraction is multiplied by an integer by multiplying the numerator by that integer.**

Also that **the product of two or more fractions is a fraction whose numerator is the product of the given numerators and whose denominator is the product of the given denominators.** Cancel when possible.

Thus, $\frac{3}{4}$ of $\frac{5}{6} = \frac{\cancel{3} \times 5}{4 \times \cancel{6}_{2}} = \frac{5}{8}$.

Oral.

1. $\frac{2}{3} \times 6 = ?$ $\frac{5}{6} \times 4 = ?$ $\frac{3}{5} \times 5 = ?$ $\frac{7}{8} \times 12 = ?$

2. $\frac{1}{3}$ of $\frac{1}{2} = ?$ $\frac{2}{3}$ of $\frac{1}{2} = ?$ $\frac{1}{6}$ of $\frac{3}{4} = ?$ $\frac{3}{4}$ of $\frac{1}{6} = ?$

Written.

Find the product of:

3. $\frac{1}{2}$, $\frac{5}{6}$.

4. $\frac{2}{3}$, $\frac{4}{5}$.

5. $\frac{5}{12}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{5}{8}$.

6. $\frac{3}{16}$, $\frac{4}{5}$.

7. $\frac{1}{2}$, $\frac{7}{12}$.

8. $\frac{5}{2}$, $\frac{8}{15}$, $\frac{1}{3}$.

Multiply:

9. 17 bu. 30 qt. × 3

10. 17 gal. 3 qt. × 1.5

11. 17 ft. 9 in. × 6.5

12. 12 mi. 95 yd. × 40

13. 100 yd. 2 ft. × $\frac{3}{4}$

14. 40.6 pk. 1 pt. × 35

NOTE.—The teacher will supply similar exercises, if needed. Some or all of pages 15–17, 35, 36, 43, may also be reviewed.

1. What is the cost of 468 books at 15¢ each?

PLAN. 1. $15¢ = 1\frac{1}{2} \times 10¢$. 2. $468 \times 1\frac{1}{2} \times 10¢ = ?$

2. What part of a number is added to the number to multiply it by $1\frac{1}{2}$? How is a number multiplied by 10?

3. What is the cost of 450 handkerchiefs at $13\frac{1}{3}¢$ each?

PLAN. 1. $13\frac{1}{3}¢ = 1\frac{1}{3} \times 10¢$. 2. $450 \times 1\frac{1}{3} \times 10¢ = ?$

4. What part of a number is added to the number to multiply it by $1\frac{1}{3}$?

Tell why the following are true, with any number as multiplicand:

TO MULTIPLY BY:	ADD TO MULTIPLICAND:	MOVE DECIMAL POINT:
5. 15	$\frac{1}{2}$ of itself.	1 place to the right.
6. $13\frac{1}{3}$	$\frac{1}{3}$ " "	1 " " " "
7. $133\frac{1}{3}$	$\frac{1}{3}$ " "	2 places " "
8. $112\frac{1}{2}$	$\frac{1}{8}$ " "	2 " " " "
TO MULTIPLY BY:	SUBTRACT FROM MULTIPLICAND:	MOVE DECIMAL POINT:
9. $87\frac{1}{2}$	$\frac{1}{8}$ of itself.	2 places to the right.
10. $66\frac{2}{3}$	$\frac{1}{3}$ " "	2 " " " "

Written.

What is the cost of:

11. 96 yd. of silk at $\$1.12\frac{1}{2}$ per yard?
12. 56 yd. of carpet at $87\frac{1}{2}¢$ per yard?
13. 72 bu. of potatoes at $66\frac{2}{3}¢$ per bushel?
14. 66 yd. of ribbon at $13\frac{1}{3}¢$ per yard?
15. 45 yd. of lace at 15¢ per yard?
16. 28 yd. of velvet at $\$1.33\frac{1}{3}$ per yard?
17. 14 yd. of gingham at $13\frac{1}{3}¢$ per yard?
18. 64 screw drivers at $87\frac{1}{2}¢$ each?
19. 25 rolls of wall paper at $87\frac{1}{2}¢$ per roll?

The product of the two numbers represented by a and b has already been denoted by ab.

Thus, if $a = 2$ and $b = 3$, $ab = 2 \times 3$, or 6. Similarly, $4a = 4 \times 2$, or 8.

Oral.

1. At c cents a quart, what is the cost of 7 qt.? Of q qt.? Of 1 gal.? Of 5 gal.? Of g gal.?

2. How many quarts are there in b bushels? How many in c crates of 16 qt. each?

3. A cubic foot of water weighs 62.5 lb.; what is the weight of c cubic feet?

4. Gold is 19.3 times as heavy as water; indicate the weight of t cubic feet of gold.

5. Indicate the cost of 10 books at d dollars each; of b books at the same price.

Order of Factors. In a product it is customary to write the numbers first, then the letters in alphabetical order.

Thus, the product of d, c, a, 2 is written $2acd$.

Written.

Write the products in each of the following:

6. 3, a. 7. 5, h. 8. d, 6. 9. x, y.

10. p, q. 11. a, 4, b. 12. b, 5, x. 13. 4, x, c, 2.

14. 5, x, $\frac{1}{2}$. 15. t, a, x, 16. 16. $3y$, b, x, c.

17. Using the following values for the letters find the value of each product in Exercises 6 to 16.

$a = 2$	$c = 6$	$h = 1$	$q = \frac{1}{14}$	$x = \frac{1}{4}$
$b = 1$	$d = \frac{1}{2}$	$p = 49$	$t = 9$	$y = 6$.

Multiply:

18. $3a + 5d + c$ times 6

19. $3a + 5b + \frac{1}{3}d$ times 3

20. $7m + p + q$ times 1.5

Summary of Division. In the ten examples of division shown at the right the principle is the same.

Each denomination is divided separately, the largest first, and the remainder, if any, is expressed in terms of the next lower denomination and united with it before that denomination is divided.

The fundamental relation of division is: Divisor × Quotient = Dividend.

The divisor may be an abstract number; in that case the quotient is of the same denomination as the dividend. If the divisor is not abstract, it must be of the same denomination as the dividend, and in this case the quotient is an abstract number.

(1)
$$3\overline{)69}\quad 23$$

(2)
$$3\overline{)6.9}\quad 2.3$$

(3)
$$\tfrac{6}{13} \div 3 = \tfrac{2}{13}$$

(4)
6 ft. 9 in. ÷ 3 = 2 ft. 3 in.

(5)
$$(6a+9b) \div 3 = 2a+3b$$

(6)
69 yd. ÷ 3 yd. = 23

(7)
$$69a \div 3a = 23$$

(8)
$$3\overline{)78}\quad 26$$

(9)
$$3\overline{)7 \text{ lb. } 8 \text{ oz.}}\quad 2 \text{ lb. } 8 \text{ oz.}$$

(10)
$$7\tfrac{8}{13} \div 3 = 2\tfrac{7}{13}$$

Oral.

1. Why are the digits of the answers the same in examples (1) to (7), but different in examples (8) to (10)?

2. How is division tested?

3. State what values must be given to a and b, in order that example (5) may in turn be the same as examples (1) to (4).

4. In which examples is the divisor abstract? What is the denomination of the quotient in these examples?

5. In which examples are the divisors concrete? What is the character of the quotient in these examples?

6. Why is there no a in the result of example 7? Give a the value 2 and perform the division.

Written.

Divide and test:

1. 987 by 31. 2. 1763 by 17. 3. 16525 by 25.
4. 2846 by 24. 5. 96064 by 32. 6. 14641 by 11.
7. 1599 by 39. 8. 6334 by 31. 9. 28346 by 29.
10. 3216.84 by 16. 11. 7683 by 29. 12. 186.75 by 25.
13. 96048 by 48. 14. 6.25 by 1.25. 15. 70.185 by 3.5.

A fraction may be divided by an integer by dividing the numerator or by multiplying the denominator by that integer.

An integer or a fraction may be divided by a fraction by inverting the divisor and multiplying.

Oral.

16. $\frac{2}{3} \div 2 = ?$ $\frac{4}{5} \div 2 = ?$ $\frac{9}{16} \div 3 = ?$ $\frac{10}{12} \div 5 = ?$
17. $2 \div \frac{1}{2} = ?$ $4 \div \frac{1}{3} = ?$ $5 \div \frac{1}{4} = ?$ $5 \div \frac{3}{4} = ?$
18. $\frac{1}{3} \div \frac{1}{2} = ?$ $\frac{2}{3} \div \frac{1}{2} = ?$ $\frac{3}{4} \div \frac{1}{3} = ?$ $\frac{3}{4} \div \frac{2}{3} = ?$

Divide and test:

19. $\frac{2}{3} \div 4$. 20. $\frac{3}{5} \div 8$. 21. $\frac{9}{10} \div 6$. 22. $\frac{5}{8} \div 12$.
23. $6 \div \frac{2}{3}$. 24. $5 \div \frac{3}{4}$. 25. $12 \div \frac{3}{8}$. 26. $20 \div \frac{11}{12}$.
27. $\frac{3}{4} \div \frac{2}{3}$. 28. $\frac{9}{10} \div \frac{5}{8}$. 29. $\frac{5}{6} \div \frac{9}{10}$. 30. $\frac{11}{12} \div \frac{7}{8}$.

Written.

31. 10 bu. 3 pk. ÷ 1 bu. 5 pk.

32. 25 ft. 8 in. ÷ 4 ft. 2 in.

33. 84 mi. 100 ft. ÷ 20.

34. 40 hr. 20 min. ÷ 15.

Year.	Total Consumption.	Consumption of wool per capita.
1840....	76,710,602	4.49
1850....	129,354,978	5.58
1860....	213,832,799	6.80
1870....	305,770,597	7.93
1880....	427,184,530	8.52
1890....	548,167,332	8.75
1900....	483,865,236	6.34

35. By use of this table find to the nearest million the number of consumers of wool in United States for each year mentioned.

NOTE.—The teacher may supply similar exercises as needed. Some or all of pages 18–23, 37–39, 43, 44, may also be reviewed.

Written.

1. How many sewing machines at $15 each can be bought for $285?

PLAN. 1. $15 = \frac{3}{5}$ of —. 2. $\$285 \div (\frac{3}{5} \times 10) = \$\frac{285}{10} \times \frac{5}{3} = —.$

2. What part of a number taken from it leaves $\frac{2}{3}$ of the number? How is a number divided by 10?

3. How many yards of silk at $\$1.12\frac{1}{2}$ a yard can be bought for $900?

PLAN. 1. $112\frac{1}{2} = \frac{9}{8}$ of 100¢ or $1. 2. $\$900 \div \frac{9}{8} = \$900 \times \frac{8}{9} = —.$

4. What part of a number taken from it leaves $\frac{8}{9}$ of the number?

Tell why the following are true, with any number as dividend:

TO DIVIDE BY:	SUBTRACT FROM DIVIDEND:	MOVE DECIMAL POINT:
5. 15	$\frac{1}{3}$ of itself.	1 place to the left.
6. $13\frac{1}{3}$	$\frac{1}{4}$ " "	1 " " "
7. $133\frac{1}{3}$	$\frac{1}{4}$ " "	2 places to the left.
8. $112\frac{1}{2}$	$\frac{1}{9}$ " "	2 " " "
TO DIVIDE BY:	ADD TO DIVIDEND:	MOVE DECIMAL POINT:
9. $87\frac{1}{2}$	$\frac{1}{7}$ of itself.	2 places to the left.
10. $66\frac{2}{3}$	$\frac{1}{2}$ " "	2 " " "

How many of each article can be bought for the amount given?

11. Yards of carpet at $\$.87\frac{1}{2}$ per yard	$70.
12. Atomizers at $33\frac{1}{3}$¢ each	$40.
13. Concert phonographs at $\$66\frac{2}{3}$	$600.
14. Parlor phonographs at $\$12\frac{1}{2}$ each	$350.
15. Teakettles at $\$1.33\frac{1}{3}$ each	$60.
16. Bookcase at $15 each	$855.
17. Pairs of dumb-bells at $\$1.12\frac{1}{2}$ per pair	$20.25.
18. Tennis rackets at $\$1.33\frac{1}{3}$ each	$24.00.
19. Tents at $\$13\frac{1}{3}$ each	$120.

The quotient of two numbers, a and b, has already been denoted by $\frac{a}{b}$.

Thus, if $a = 2$, $b = 3$, then $a \div b = \frac{a}{b} = \frac{2}{3}$. Similarly, the quotient of $6a$ and 3 is $\frac{6a}{3} = 2a$.

Oral.

1. Find the quotient of $10a \div 5$; of $4a \div 2a$.

Find the quotients:

	2.	**3.**	**4.**	**5.**	**6.**	**7.**
Dividend:	16	$16a$	$36c$	$3b$	$5xy$	$12mp$
Divisor:	2	2	$9c$	b	y	$2m$

8. 10 books of equal weight together weigh p pounds; what is the weight of each book? If n such books weigh 25 lb., what is the weight of each? If n books weigh p pounds, what is the weight of each?

9. The daily receipts of a hotel charging \$2 a day are \$400; how many guests are there? How many guests are there if the daily receipts are $8x$ dollars?

10. A railway expects to have $3r$ passengers on an excursion; if its cars hold 60 passengers each, of how many cars must the train consist?

Written.

Write the quotient of the following pairs of numbers, the first being the dividend in each case:

11. 20, 7. **12.** $\frac{11}{4}$, $\frac{9}{4}$. **13.** $6a$, 3. **14.** 5, b.

15. p, q. **16.** $8x$, y. **17.** m, $2a$. **18.** $3r$, $5q$.

19. Find the value of each result in Exercises **11** to **18**, using the following values of the letters:

$a = 6$.	$m = 9$.	$q = 5$.	$x = 11$.
$b = 12$.	$p = 8$.	$r = 7$.	$y = 15$.

Find the quotients:

20. $(9a + 15x) \div 3$. **21.** $(15m + 35) \div 5$.

22. $(21 + 49y) \div 7$. **23.** $(16t + 8v) \div 4$.

24. $(8l + 32) \div 8$. **25.** $(10x + 90) \div 5$.

XIII

APPLICATIONS OF PROCESSES

PROBLEMS—FORESTS AND LUMBER

Written.

1. The woodland area of the United States is about 35% of the total area, 3,025,600 sq. mi.; how many square miles are woodland?

2. How many acres of woodland are there in the United States? (640 acres = 1 sq. mi.)

3. The government and many states have established forest preserves as follows; find the total acreage:

YEARS.	ACRES.	YEARS.	ACRES.
1891.....	13,457,680	1898–1901....	7,050,089
1893.....	4,443,000	1902–1904....	15,995,196

4. In 1900 there were 33,035 establishments manufacturing from lumber $611,611,524 worth of products; what was the average output of each factory?

5. These factories employed 283,260 workmen at an average wage of $370 a year; find their total earnings.

6. In 1900 the lumber mills sawed 35,084,166,000 board feet, valued at $566,832,984; this was an increase of 29% over the value of the product in 1890; what was that value?

7. It is estimated that trees containing 21,239,000,000 ft. of spruce are now standing in Maine and increasing at the rate of 600 million feet a year; if none are destroyed, how many feet would there be in 25 years?

8. Taking the annual capacity of the Maine pulp mills to be 275 million feet, how much may be used for other purposes without exceeding the natural increase of the timber?

One of the greatest railway undertakings of our day is the *Cape-to-Cairo Railway.*

Written.

1. Find from the map the length of this projected railway from Cairo to Cape Town.

2. How many miles are completed?

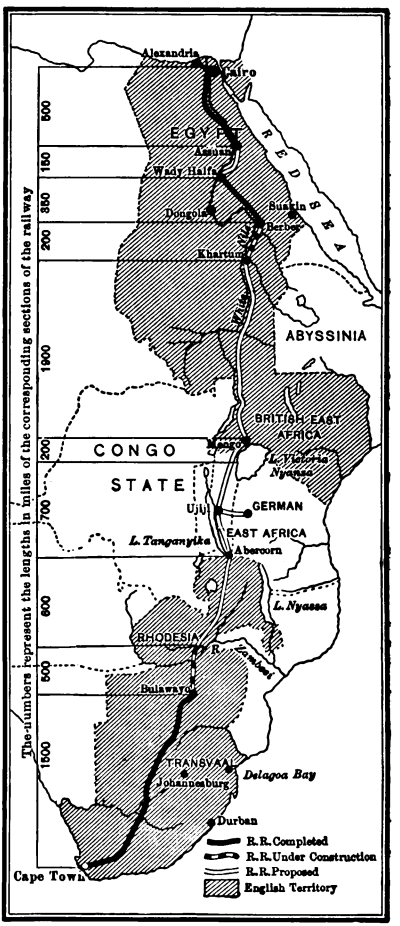
3. How many miles are under construction?

4. Altogether 3,000 miles are already built throughout the completed and partially built sections, $\frac{2}{3}$ at a cost of $15,000 a mile and the rest at $19,000; what has this part of the line cost?

5. If the remainder of the line cost $75,000,000, how much is this per mile? What will the whole line cost per mile?

6. The shaded portion of the map represents English territory; how many miles of foreign territory would the railway traverse?

7. At a speed of 30 mi. an hour, how long would it take a train to go from Cairo to Cape Town?



8. Find the fare from Cape Town to Cairo at 3¢ a mile.

Oral.

1. A large cattle ranch in Texas contains 700,000 acres; what is the land worth at $7 an acre?

2. A ranch had 100,000 cattle; its owner employed 300 cowboys; how many was this for every 1,000 cattle?

3. 1,200 ponies were used in herding the cattle; how many was this for each man?

4. For grazing, cattle are grouped in herds of from 125 to 200 each; how many herds are there in 100,000 cattle when the smallest number is grouped in each herd? When the largest number is taken?

5. What does it cost to pay the cowboys for one year at $40 a month?

6. What would 1,200 ponies cost at $25 each?

7. A herd of cattle traveling 15 miles a day was driven $37\frac{1}{2}$ miles to the railroad; how many days did it take?

8. How many cattle cars carrying 18 steers each are needed to carry 3,600 steers?

9. At 150 mi. a day, how many days does it take the train to reach Omaha, 500 miles distant?

10. The cattle averaged 800 lb. in weight and sold at 5¢ a pound; how much did one animal bring?

11. For how many dollars did the 3,600 steers sell?

12. On a ranch in Kansas a blizzard froze 5% of a herd of 15,000 cattle; how many were frozen?

13. About 20 million dollars' worth of beef is exported annually; of this the British Isles take 99%; what is the value of the beef exported to all the rest of the world?

14. Our total exports of canned beef are worth $5,000,000; of which Germany takes 10% and the British Isles 65%; what is the value of the canned beef exported to each?

Written.

The coffee berry grows on a shrub from 4 to 6 ft. in height. The average yield of a coffee tree is 6 lb. of salable coffee in one season. The average productive life of a tree is 30 years.

1. Find the production of 1 coffee tree during its life.

2. What is the coffee worth at $15\frac{3}{4}¢$ a pound?

3. One acre of land grows about 800 trees; what would be the yield of coffee on 1 acre in one season?

4. What would be the yield of a plantation of 600 acres?

5. What would be the value of this yield at $14\frac{1}{4}¢$ a pound?

6. Different countries produce different grades of coffee:

COUNTRY.	WHOLESALE PRICE.	COUNTRY.	WHOLESALE PRICE.
Mexico.....	$15\frac{3}{4}¢$	Venezuela.....	$13\frac{1}{2}¢$
Costa Rica	$14\frac{1}{4}$	Colombia.....	$13\frac{1}{2}$
Guatemala.....	$14\frac{1}{2}$	Brazil.....	$9\frac{1}{4}$

What would the product of the plantation mentioned in Exercise 4 be worth at the price named for each country?

7. The coffee crop of Brazil often amounts to 11,000,000 sacks of 132 lb. each; how many pounds is this?

8. What is the value of this product according to Exercise 6?

9. 70 lb. of coffee occupy about 1 cu. ft.; how many pounds are there in the bed of dried coffee shown in the picture?

10. Venezuela averages 55,000 tons of coffee a year, $\frac{9}{10}$ of which is exported; what is the value of the exports according to Exercise 6?

Oral.

1. How many trees are there in an orange grove of 100 acres, averaging 30 trees to the acre?

2. Nine years are required to bring an orange grove to maturity. If the average annual expense of care for the grove was \$10 an acre, what was the expense of raising the grove of Exercise 1?

3. If the trees yield an average of 20 boxes per season, how many boxes are there in a single crop from this grove?

4. If the boxes average 128 oranges at $\frac{7}{8}¢$ each, what is the value per box?

5. The cost of picking, grading, and packing is 8¢ a box; what would be the cost of this work for one crop from this grove?

6. If these oranges are sold for \$1.25 a box, ready for shipment, what does the grower receive for the crop?

7. At 90¢ a box what are the express charges on the crop?

8. Allowing 10¢ a box for other expenses, for what must the wholesaler who pays the expressage sell the crop to gain 50¢ a box?

Written.

9. If the retailer pays \$2.75 per box of 128 oranges and sells the oranges at 35¢ a dozen, what does he make on a box?

10. If the retailer buys larger oranges, 96 to a box, at the same price and sells them at 50¢ per dozen, how much does he make per box?

11. According to Exercises 9 and 10, on which kind of oranges does the retailer make the more? How much per hundred boxes?

12. According to Exercises 1, 3, and 4, how many oranges are there in a season's crop?

Written.

1. A farmer kept the following account of a field of corn:

EXPENSES PER ACRE.				RETURNS.	
Plowing.....	$2.50	Seeding.....	$1.65	50 bu. at...	$.45
Harrowing	2.50	Cultivating....	4.50	Fodder.....	8.00
Fertilizing.....	.80	Harvesting	8.00		

Find the total returns, expenses, and net profit per acre. What was the net profit from 25 acres of corn?

2. All the street railways of New York City carried 1,036,000,000 passengers in a recent year. Taking the population as 3,400,000 what was the average number of times that each inhabitant used the street railways?

Add:

3. $2a + 3b$ / $5a + 6b$

4. $2c + 5n$ / $3c + 8n$

5. $\frac{1}{3}a + \frac{1}{6}b$ / $\frac{2}{3}a + \frac{5}{6}b$

Subtract:

6. $5a + 8b$ / $2a + 5b$

7. $.9a + .05d$ / $.3a + .01d$

8. $\frac{2}{3}b + \frac{4}{5}c$ / $\frac{1}{3}b + \frac{1}{10}c$

Multiply:

9. $3, 4a, c$.

10. $x, y, 2z$.

11. $m, n, 3p$.

12. $4x, a, y$.

13. What is the cost of 7 yd. of cloth at $\$1.33\frac{1}{3}$ a yard?

14. A Washington lumber mill cuts 87 thousand feet of lumber every 10 hours; how many feet will it cut in 26 days of 10 hours each? If an ordinary 10-room house requires 7 thousand feet of lumber, how many such houses will one day's product build?

15. Some orange groves net their owners $875 an acre; at this rate what is the profit on $55\frac{1}{2}$ acres?

16. The value of the furniture manufactured annually in this country is $80,000,000. There are about 1,500 factories; what is the average output for each factory?

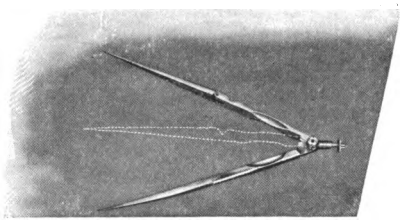
XIV

FORM STUDY AND MEASUREMENT

ANGLES

Instruments: Rule, protractor and compasses

1. Lay the compasses down flat, as shown in the picture. The two arms form an angle. Move one arm of the compasses from one position to another. How is the size of the angle changed? Does the change depend upon the length of the arms of the compasses?



2. Draw two lines forming an angle.

3. Draw three lines forming two angles arranged as in Figure 1. If the angles are 20° and 25°, what is their sum?

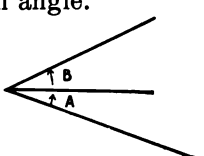


Fig. 1.

4. Draw two other angles; measure them with a protractor and find their sum.

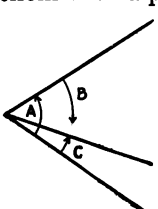


Fig. 2.

5. Two angles, as A and B of Figure 2, are 65° and 50°. The equation A − B = C means that C is the difference between A and B. How many degrees in C?

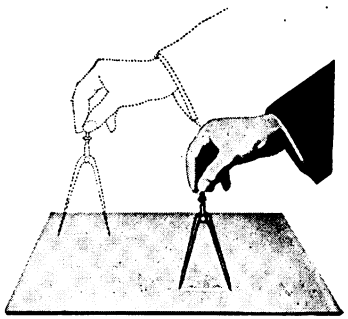
6. Draw a figure like that shown, but larger. Measure the angles A and B. Find C. Verify by measurement.

Draw the following angles with the protractor, then construct the sum of each pair; the difference between each pair:

7. 85°, 25°. **8.** 50°, 20°. **9.** 120°, 60°.

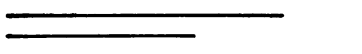
10. 80°, 45°. **11.** 70°, 55°. **12.** 130°, 10°.

1. Draw a straight line $2\frac{1}{2}$ in. long. Place the points of the compasses at its end points. Transfer the points of the compasses to another position. How far apart are the new points?



2. Place the points of the compasses against the edge of a rule, and by means of the spaces spread the points 3 in. apart. Transfer the points of the compasses to paper. Connect the points. What is the length of the line?

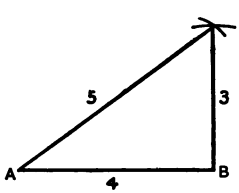
3. Transfer these lines from the book to your paper:



4. Draw a line of convenient length, and using the length of the line as radius, draw an arc with each end as center. Connect the point of cutting with the ends of the line. What kind of a figure is formed? How do its sides compare?

5. A lot is in the shape of an equilateral triangle of side 300 ft. Draw a plan of the lot, taking 1 in. to represent 100 ft.

6. Draw a line 3 in. long. About each end as a center draw a circle of radius 5 in. Connect an intersection of these circles with the ends of the first line.



7. How many sides of the triangle of Exercise 6 are equal? It is an *isosceles* triangle.

8. State a method of drawing an isosceles triangle with sides of given length.

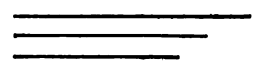
9. Draw a line 4 in. long corresponding to A B in the figure; by drawing arcs, find a point that is 3 in. from B and five in from A.

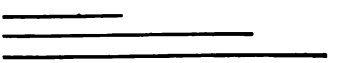
1. Draw a straight line 4 in. long. Find points that are at once 3 in. from one end and $2\frac{1}{2}$ in. from the other.

2. Draw a triangle whose sides are 4 in., 7 in., and 8 in.

3. Tell how to draw any triangle, given its three sides.

Construct with compasses triangles having the following sides:

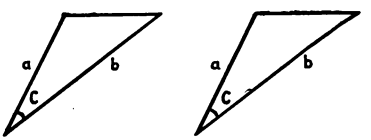
4. 

5. 

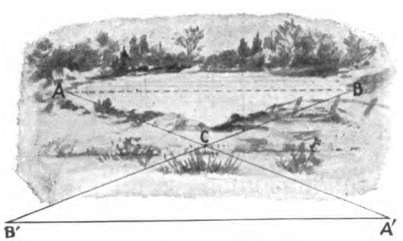
6. Draw two triangles, the sides of each being 3 in., 5 in., and 6 in. Cut out the triangles and apply one to the other. Can they be made to coincide exactly?

7. If three sides of a triangle are respectively equal to three sides of another, how do the triangles compare?

8. Draw two triangles with parts equal as shown in the picture. Cut them out. Can they be made to coincide?

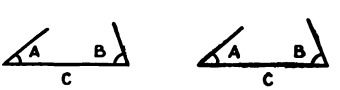


9. If the two sides and included angle of a triangle are respectively equal to the two sides and the included angle of another triangle, how do they compare in size and shape?



10. To measure the length, A B, a surveyor measured A C and C B. He laid off a distance equal to A C (Where?), and one equal to B C (Where?), and measured the line, A′ B′. Why was it equal to A B?

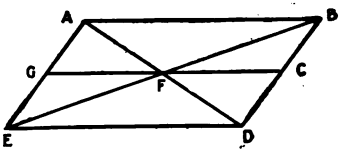
11. Draw two triangles with the parts equal that are shown in the figure. Cut them out and apply one to the other.



12. If two triangles have two angles and the included side of one equal to two angles and the included side of the other, how do they compare?

Oral.

In the figure what angle is the sum of angles:



1. C F B and D F C?
2. B F A and A F G?
3. A B F and F B C?

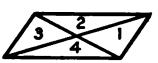
Of what angles is each of the following the difference?

4. D F C.
5. B F A.
6. C F B.
7. F D E.
8. E A D.
9. E G F.

Written.

10. If convenient, test these relations by drawing a figure and measuring the angles.

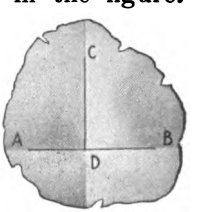
11. Draw two parallel lines each 4 in. long, and connect their ends so as to form a parallelogram shaped like the figure.



12. Draw the diagonals, or lines joining the opposite corners and number the triangles in order.

13. Cut out the figure just mentioned, and cut along the diagonals. Select the equal triangles. Mark the equal angles and equal lines.

14. Place the pieces together again to form the parallelogram. How does the intersection of the diagonals divide each one? Point out sets of equal angles in the figure. Find other equal angles by addition.

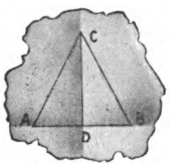


15. In any piece of paper, preferably irregular, as shown in the figure, fold a straight line, A B. Bisect A B by folding A to B. This makes the crease C D. Angle B D A is a **straight angle,** or an angle of 180°.

16. Compare angle B D C and C D A by folding. How many degrees are there in each? Two lines which make a **right angle** (90°) with each other are said to be **perpendicular.**

17. How is line C D related to the line A B?

1. Fold a line, A B, and the line perpendicular to it at its middle point, as C D in the figure.



2. Fold or draw lines from C to both A and B. Compare the length of C A with C B.

3. Repeat for other points on C D.

4. What sort of triangle is A B C? Why?

5. When the figure is folded along C D, how are triangles D B C and D A C placed? How do they compare?

6. How do the angles opposite the equal sides of an isosceles triangle compare in size?

7. How does the line C D divide the angle A C B?

8. How does the altitude of an isosceles triangle divide the base? The vertical angle? The triangle?

9. Answer Exercise 8 for an equilateral triangle.

10. It has been shown (Book II, p. 216) that the sum of the 3 angles of a triangle is 180°; when one angle is given how may the sum of the other two be found?

11. If the angle between the equal sides of an isosceles triangle is given, how may the other angles be found?

12. How many degrees in each angle a in Fig. 1?

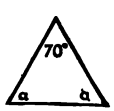


Fig. 1.

13. Draw an isosceles triangle with base 3 in. and base angles 45°. How many degrees in the angle at the vertex?

14. Draw an isosceles triangle with a base of 2 in. and an angle at the vertex of 40°.

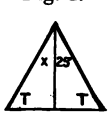


Fig. 2.

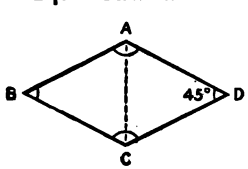
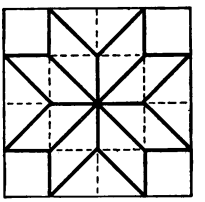


Fig. 3.

15. Find the number of degrees in the angles x, T, and T in Fig. 2.

16. Find the number of degrees in the angles A, C, and B, the parts of Fig. 3 being equal isosceles triangles.

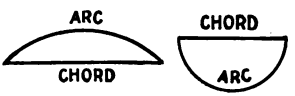
The *square, triangle, rectangle* and *parallelogram* are the basis of many decorative designs.



1. Draw a square 2 in. on a side. Divide each side into four equal parts. Join the corresponding points in the opposite sides by dotted lines. Draw heavy lines, as in the figure. Erase the dotted lines. Shade in two ways or color in two colors.

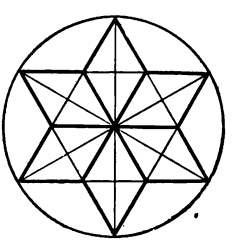
2. Draw an arc of a circle and connect its ends by a straight line.

Chord. The straight line connecting the two ends of any arc is called the *chord* of that arc. The chord is said to *subtend* the arc.

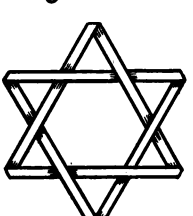


3. What is the longest chord in any given circle?

4. Draw a circle. By means of the protractor draw diameters making angles of 60°. Connect the ends of these diameters alternately by chords, as shown in the figure. Join the center with the intersections of the chords. Erase all lines except those corresponding to the heavy lines of the figure. A six-pointed star results. Shade in two ways or color in two colors.



5. Draw a square; bisect the sides; connect these points in order about the square. Color or shade the triangular parts.



6. Fold or weave two narrow strips of paper of equal length in the form shown at the left; of what two figures is it composed? If the side of one of these is 2 in. long, how many inches of paper are needed to make the design?

1. What is a six-sided plane figure called?

The regular hexagon occurs frequently in nature, and is much used in architecture and designs. It is constructed by drawing a circle and drawing successive chords equal to the radius. (See Figure 1.)

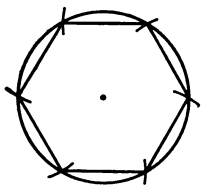


Fig. 1.

2. Construct a regular hexagon using dotted lines for the sides.

3. From the center of the figure draw full lines to the vertices. Along each such line draw short parallel lines as shown in Figure 2. The resulting figure is a common pattern among snow crystals.

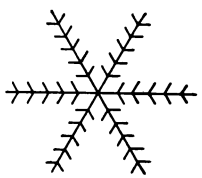


Fig. 2.

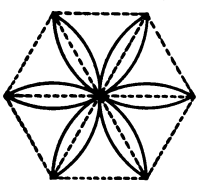


Fig. 3.

4. Draw a regular hexagon in dotted lines. By using each vertex as a center, describe the small arcs shown in Figure 3. Complete the figure and erase the dotted lines.

5. Show how to draw the designs of these figures:

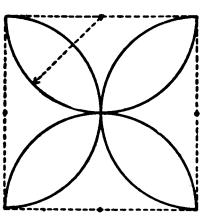


Fig. 4.

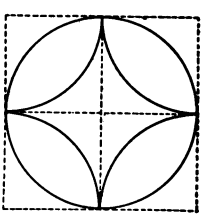


Fig. 5.

6. Explain from the figures how to construct a design for each of these moldings:

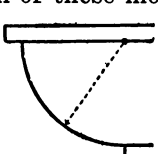


Fig. 6.

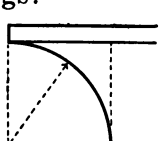


Fig. 7.

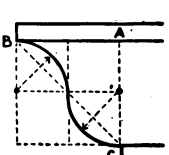


Fig. 8.

Oral.

1. What is an angle? Illustrate your answer.

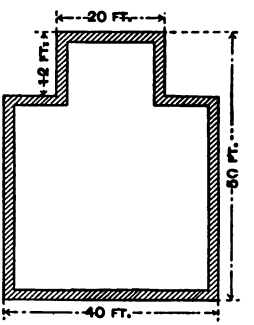
2. Explain how to construct a triangle whose sides shall be equal in length to three given lines; also how to construct an equilateral triangle having its sides equal to a given line.

3. What is an isosceles triangle? If the angle at the vertex is 80°, what is the size of each angle at the base?

4. What is the sum of the angles of a triangle? When the three angles of a triangle are equal, how many degrees in each?

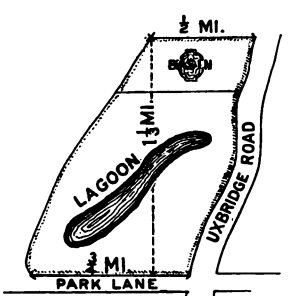
5. If the three sides of one triangle are respectively equal to the three sides of another triangle, how are the triangles related? How if two sides and the included angle of one are respectively equal to two sides and the included angle of the other?

6. Two points are 8 in. apart; how may a point be found that is 10 in. from one of these points and 5 in. from the other?



Written.

7. The figure shows the plan of a basement wall. How many cubic yards of stone will be required to build the wall 8 ft. high and 18 in. thick?



8. The main hall of the Providence railway station is 180 ft. long and 44 ft. wide; what is the area of the floor?

9. The train shed of the same station is 588 ft. long and 130 ft. wide; what area does it cover?

10. The figure shows the general shape and size of Kensington Gardens, Hyde Park, London; regarding it as a trapezoid, find its area.

XV

SOLUTION OF PROBLEMS

ANALYSIS

How to Solve Problems. The four steps in the solution of a problem have already been explained and applied (Book II, pages 92, 93), namely:

1. **Read the problem.** 2. **Plan the solution.**
3. **Make the calculation.** 4. **Test the work.**

The analysis of a problem is the plan of the solution. It may be expressed orally or written in steps.

EXAMPLE. A dealer buys Welsbach lamps at $9.00 a dozen; for how much must he sell them to gain 50 cents each?

ANALYSIS.	CALCULATION.
1. $9.00 ÷ 12 = $.75.	$\begin{array}{r} .75 \\ 12\overline{)9.00} \end{array}$ + $.50 = $1.25.
2. $.75 + $.50 = $1.25.	

Oral Analysis. $\frac{1}{12}$ of $9.00, or $.75, is the cost of 1 lamp.

$.75, the cost, plus $.50, the gain, is $1.25, the selling price of the lamp.

Solve these problems, writing the analysis in step form and the calculation; also give the oral analysis:

1. A man bought a hat for $2.50 and 6 collars at 15¢ each; how much did he pay for these articles?

2. $3\frac{1}{2}$ lb. of coffee cost $1.12; what was the cost 1 lb.?

3. A dealer paid $30 for a dozen pairs of shoes; he sold them at a gain of $.75 a pair; find the selling price a pair?

4. Taking 500 bunches of lath as a carload, how many carloads would 3,500,000 bunches make?

5. How many times would the water in a cistern containing 5,180 gal. fill a bucket that holds $2\frac{1}{2}$ gal.?

NOTE.—The oral exercises provide opportunity throughout the book for drill in oral analysis.

Important Methods. There is no single plan that will apply to the solution of all problems of arithmetic. But there are two methods widely used, **unitary analysis** and the **equation.** We have already used both methods many times; they will be reviewed here.

Unitary Analysis. Unitary analysis may best be explained by a few examples:

(1) What is the cost of 45 bottles of ink at $3.00 per hundred?

ANALYSIS.

1. 100 bottles of ink cost $3.00.
2. Therefore, 1 bottle costs $\frac{1}{100}$ of $3.00, or $.03.
3. Therefore, 45 bottles cost 45 × $.03, or $1.35.

(2) If 14 bu. of peaches make 270 qt. of fruit for canning, how many qt. will 63 bu. make?

ANALYSIS.

1. 14 bu. make 270 qt.
2. Therefore, 1 bu. makes $\frac{1}{14}$ of 270 qt., or $19\frac{5}{7}$ qt.
3. Therefore, 63 bu. make 63 × $19\frac{5}{7}$ qt., or 1215 qt.

It is usually better first to indicate all the work, then cancel as much as possible, and multiply last of all.

The solution of example (2) would appear thus:

$$\overset{1}{\cancel{14}}\ \overset{3}{\cancel{270}}\ \overset{5}{\cancel{\ }} \times \overset{9}{\cancel{63}} = 1215$$

$$\cancel{\ }\ \cancel{\ }\ \cancel{\ }$$

Written.

1. If 25 yd. of cloth make 15 vests, how many yards are needed to make 36 vests?

2. If a steamer goes 9 mi. in 48 minutes, how long will it take to go 51 mi. at the same rate?

3. If the interest on $330 is $22, how much is the interest on $780 for the same time and rate?

4. If 16 men can dig 980 yd. of sewer trench in 8 days, indicate the amount they can dig in 1 day; the amount 1 man can dig in 1 day; in 20 days; the amount that 28 men can dig in 20 days. Find the last named amount.

Oral.

1. If r represents the speed of a train per second, what represents its speed per minute? Per hour?

2. If R represents the speed per hour, what represents the speed per minute? Per second?

3. If s represents the speed of a train per minute, what represents the distance the train will travel at that rate in 10 minutes? In an hour? In 5 hours? In h hours?

4. How many seconds will a bicyclist require to ride 3,000 ft. at the rate of 15 ft. a second? At the rate of r ft. a second? How many seconds will he require to ride s feet at the rate of r feet a second?

5. If l, b, and h represent respectively the number of feet in the length, breadth, and height of a room, what represents the number of cubic feet in the room?

6. If c represents the number of cubic feet of pure air required every hour by one person, what represents the number of hours that the quantity of air contained in this room would supply one person? 10 persons? p persons?

7. When x is the age of one man and y that of another, what does $x + y$ stand for? $x - y$? $2x + y$? $x - 2y$? $\frac{1}{2}x + \frac{1}{2}y$, that is $\frac{1}{2}$ of $x + \frac{1}{2}$ of y?

Instead of the expression "$x + y$ stands for" or "n stands for" we may use "$x + y$ is," or "n is."

8. What does $x - y = 7$ mean? $2x + y = 137$? $\frac{2}{3}x + 3y = 155$?

9. If H is the number of dollars a house costs, and L the number of dollars the lot costs, what is H + L? What is H − L? What does H = L mean? H = 2L? H = L + 500? H = L − 500?

10. If l is the length of a rectangle and b is its breadth, what is lb? What is $2l + 2b$? What does $l = 2b$ mean? What does $l = b + 20$ mean? $b = l - 10$?

Oral.

1. One side of a balance contains $(x + 2)$ ounces and the other $(2 + 8)$ ounces; state the equation which expresses that these two scale pans balance. Write it on the board.

2. If 2 oz. are taken from each pan, how will the balance be affected? State the equation which expresses the resulting condition. Write it.

3. If, instead of 2 oz. being subtracted, 2 oz. had been added to both pans, how would the balance have been affected? State the equation which expresses the resulting condition. Write it.

4. If the weight in each pan is doubled, will the balance be affected? If each weight is made $\frac{1}{3}$ as large?

Show how the second equation follows from the first in each case:

5. $2x = 12$
$x = 6$

6. $x + 6 = 14$
$x = 8$

7. $x + a = a + 8$
$x = 8$

8. $x + 2 = 2 + 8$
$x + 6 = 6 + 8$

9. $x + 3 = 3 + 5$
$x + 1 = 1 + 5$

10. $x + 2a = a + 7$
$x + a = 7$

Members of Equations. The sign of equality separates an equation into two parts called **members**; that on the left is called **left** (or **first**) member, and that on the right, the **right** (or **second**) member.

11. Name the first member in each equation in Exercises 5 to 10. Name the second member.

12. x less 3 plus $3 = ?$ $x - 3 + 3 = ?$ $4x - 5 + 5 = ?$ State how $4x - 5 + 5 = 15 + 5$ becomes $4x = 20$.

Principle: If both members of an equation are increased, diminished, multiplied, or divided by the same number, the resulting members are equal.

State the operation which changes the first equation into the second in each case:

13. $x = a + 5$
$x - 5 = a$

14. $5x = 17$
$x = 3\frac{2}{5}$

15. $2x - 1 = 3$
$x = 2$

Written.

1. $\frac{3}{4}$ of the distance from Ann Arbor to Detroit is 30 mi.; find the distance between these places.

PLAN. 1. Let x be the distance. 2. Then $\frac{3}{4}x = ?$
3. $3x = ?$ Why? 4. $x = ?$ Why?
5. Test: $\frac{3}{4}$ of $40 = 30$, as the problem states.

Find the distances between the cities named in Exercises 2–4, if:

2. $\frac{4}{15}$ of the distance from Detroit to Chicago is 76 mi.

3. $\frac{7}{15}$ of the distance from Boston to Cincinnati is 441 mi.

4. $\frac{2}{3}$ of the distance from Albany to Buffalo is 198 mi.

5. Twice a certain number plus 3 is 15; find the number.

PLAN. 1. Let n represent the number; write 2 times n plus 3 in symbols.
2. Write the equation which shows that this expression equals 15.
3. What does $2n$ equal? What does n equal?
4. Test: 2 times the number found plus 3 should be 15.

Always test your results.

6. 3 times a certain number plus 6 is 12; find the number.

7. John is x years old and James is 4 years older. The sum of their ages is 26 years; how old is each boy?

8. If a tennis ball rebounds to $\frac{2}{3}$ of the height from which it was dropped, from what height must it be dropped to rebound $3\frac{1}{2}$ ft.?

9. How high will the same ball rise on the second rebound? On the third?

10. A tower and flag staff are together 100 ft. high; the staff is $\frac{1}{4}$ of the height of the tower; find the height of each.

11. A house and lot is worth \$3,500; the house is worth 6 times as much as the lot; find the value of each.

12. The part of a bridge pier under water is $\frac{2}{3}$ as high as the part out of water; the whole height is 45 ft.; find the height of each part.

Oral.

What operations are performed in finding x in each case?

1. $2x = 4ab$
   $x = 2ab$

2. $x + a = 3b$
   $x = 3b - a$

3. $x - a = 10$
   $x = 10 + a$

4. $3x = 6a + 6b$
   $x = 2a + 2b$

5. $2x + 2 = 14$
   $x = 6$

6. $3x - 5 = 7$
   $x = 4$

Written.

7. Six men hire an automobile for a day. By taking in 4 more the expense for each man is diminished \$2. What is the charge for the use of the automobile?

PLAN. 1. Let x be the charge for the use of the automobile.
2. Then $\frac{x}{6}$ would have been the cost to each of the six men.
3. $\frac{x}{10}$ is the cost to each of the 10 men.
4. $\frac{x}{6} - \frac{x}{10} =$ \$?
5. $\frac{1}{15}x =$ \$2. Why?
6. Therefore, $x =$ \$—.
7. Supply the test.

8. 5 lb. of beef and 5 lb. of lamb together cost \$1.35; the beef cost $\frac{4}{5}$ as much as the lamb; find the price of each.

9. \$1,900 was divided among 3 heirs so that the second received $\frac{3}{5}$ as much as the first, and the third $\frac{1}{2}$ as much as the second; how much did each receive?

PLAN. 1. Let x be the number of dollars received by the first.
2. Then $\frac{3}{5}x$ is the amount received by whom?
3. And $\frac{1}{2}$ of $\frac{3}{5}x$ is the amount received by whom?
4. Therefore, $x + \frac{3}{5}x + \frac{3}{10}x =$ \$— ?
5. Therefore, $x =$?
6. The first person received \$—. The second \$—. The third \$—.
7. Supply the test.

10. Divide \$4,500 into two parts such that one part is $3\frac{1}{2}$ times the other.

Solve for x and test:

11. $6x + 3 = 7$.
12. $6x + 3 = 4x + 7$.
13. $5x - 2 = 3x + 12$.
14. $10x + 5 = 9x + 8$.
15. $\frac{3}{2}x + 4 = \frac{1}{3} + 10$.
16. $\frac{4}{5}x - 1 = \frac{1}{5} + 8$.
17. $13x = 3x + 5$.
18. $\frac{2}{3}x - \frac{7}{8} = \frac{5}{12}x + \frac{2}{9}$.

Written.

1. A house and lot, sold at a gain of 10%, brought \$2,200; what was its former cost?

PLAN. 1. Let x be the cost of the property.
2. Then 110% of x, or $1.10x$, is the selling price.
3. Therefore, $1.10x = \$2{,}200$. 4. Therefore, $x = \$$—.

2. A hardware dealer sold a furnace for \$180 at a gain of 5%; what did the furnace cost him?

3. A merchant sold a damaged carpet for \$42.50 at a loss of 15%; what did the carpet cost him?

4. A collector remitted \$475 after deducting a fee of 5%; how many dollars did he collect?

5. The amount of a certain principal at 4% simple interest for 1 year was \$416; what was the principal?

6. A capital is invested thus: $\frac{1}{2}$ at $5\frac{1}{2}\%$; $\frac{1}{3}$ at 6%; and the rest at $4\frac{1}{2}\%$. The annual income is \$600; what is the capital?

PLAN. 1. Let x be the capital in dollars.
2. Then, the parts invested are $\frac{1}{2}x$, $\frac{1}{3}x$, $\frac{1}{6}x$.
3. $.055 \times \frac{1}{2}x + .06 \times \frac{1}{3}x + .045 \times \frac{1}{6}x = \$$—, the income.
4. Therefore, $.0275x + .02x + .0075x = -x = \$600$.
5. Therefore, $x = \$$—.

7. By investing $\frac{1}{4}$ of his capital at 3% and the rest at 6%, Mr. White received \$10 a year more than by investing the whole at 5%; what was his capital?

8. A man sold some of his land at \$400 an acre. He invested the proceeds at 5% and found his daily income (365 da. = 1 yr.) from the investment to be \$1.50; how many acres did he sell?

9. The price of wheat was $78\frac{1}{2}¢$ on Wednesday. It rose a certain amount on Thursday and half as much on Friday when the price was 80¢; what was the price on Thursday?

10. A merchant bought 2 canisters of coffee, 50 lb. each, for \$23.25. One kind cost $4\frac{1}{2}¢$ a pound more than the other. Find the price per pound of each.

Written.

1. In latitude 42°, about that of New York, the longest day (from sunrise to sunset) lacks 2 hr. 48 min. of being twice as long as its night (from sunset to sunrise). Find the length of each.

2. In latitude 30°, about that of New Orleans, the longest day is 3 hr. 52 min. longer than the shortest day; and twice the shortest day is 6 hr. 12 min. longer than the longest day. Find the length of each.

3. In latitude 50°, about that of Paris, the longest night is $\frac{1}{2}$ hr. more than twice as long as the shortest day. Find the length of each.

4. A traveler who has gone $\frac{1}{10}$ of the length of the St. Lawrence River from its mouth is still 1,548 miles from its source; how long is the St. Lawrence?

5. The area of Tennessee exceeds that of Kentucky by 1,650 sq. mi.; the sum of their areas is 82,450 sq. mi. Find the area of each.

6. The area of Kansas is twice that of Ohio. The sum of their areas is 123,000 sq. mi. Find the area of each.

7. The area of Florida is 9,000 sq. mi. greater than that of Pennsylvania. The sum of their areas is 99,000 sq. mi. Find the area of each.

8. The two largest states of the Union are Texas and California, and Texas exceeds California in area by 106,000 sq. mi. The sum of their areas is 418,000 sq. mi. Find the area of each.

9. If London were to increase $\frac{1}{10}$ in population, it would have 4,906,000 inhabitants; what is its population?

10. If Berlin were to increase $\frac{1}{20}$ in population, it would lack 5,000 of having 2,000,000 inhabitants. Find the population of Berlin.

The Extent of our Country. The area of the United States, excluding Alaska and the island possessions, is 3,025,600 sq. mi. Throughout this page "The area of the United States" means this area.

Written.

1. The area of the United States exceeds 50 times that of Alaska by 25,100 sq. mi.; what is the area of Alaska?

2. 84 times the area of Porto Rico is 1,600 sq. mi. less than the area of the United States; what is the area of Porto Rico?

3. If the area of the Philippine Islands were increased by 2,000 sq. mi., 20 times this area would be 125,600 sq. mi. less than the area of the United States; what is the area of the Philippines?

4. The area of all the other island possessions diminished by 1,056 sq. mi. is $\frac{1}{6}$ of the area of Porto Rico found in Exercise 2; find the area of these possessions.

5. The area of France increased by 95,908 sq. mi. is $\frac{1}{10}$ of the area of the United States when diminished by 25,600 sq. mi; what is the area of France?

6. The area of the United States diminished by 25,000 sq. mi. is $\frac{3}{4}$ of that of China when diminished by 218,401 sq. mi.; what is the area of China?

7. The total area of the United States, 3,756,884 sq. mi., increased by 243,116 sq. mi., is $\frac{1}{3}$ of the area of the British Empire when increased by 853,916 sq. mi.; find the area of the British Empire.

8. 25 times the area of the British Isles is 600 sq. mi. less than the area of the United States; find the area of the British Isles.

9. The area of Germany diminished by 8,830 sq. mi. is $\frac{1}{15}$ of the area of the United States when diminished by 25,600 sq. mi.; what is the area of Germany?

Written.

1. Twice the number of pounds of meat consumed annually per inhabitant in Great Britain is 186% of the consumption per inhabitant in the United States. If the latter is 150 lb., what is the consumption in Great Britain?

2. The number of pounds, as in the previous problem, for the United States falls 39 lb. short of being three times the number for Germany. Find the latter.

3. An average workman should eat daily a certain weight of starchy foods, 16% of that weight of fats, and 20% of that weight of albuminous food (protein). The total weight of these three foods consumed daily should be at least $1\frac{1}{2}$ lb.; what weight of each is required?

4. 18% by weight of wheat is lost (as bran, etc.) in grinding it into flour; how many 60-pound bushels of wheat are used in making 246 lb. of flour?

5. The weight of bread is $133\frac{1}{3}\%$ of the weight of the flour used to make it; according to Exercise 4 how many one-pound loaves of bread can be made from 10 bu. of wheat?

6. Chicago packs more hogs for market than any other city. In a recent year the number less 652,953 was equal to $\frac{1}{4}$ of the total for the United States, or $\frac{1}{4}$ of 25,245,000; how many hogs were packed in Chicago?

7. Cuba is the greatest cane-sugar producing country. In a recent year, it produced 980,000 tons. This was 80,000 tons more than 3 times the product of Louisiana; how many tons did Louisiana produce?

8. In the dairies of the United States the milk of some cows is used to make cheese, that of others to make butter. The average amount of butter produced annually per cow is 130 lb. This is 16 lb. less than twice the average amount of cheese. Find the latter amount.

Written.

1. From the oldest known arithmetic (Egyptian, about 1700 B.C.): "Behold the shepherd with 70 oxen. The mathematician asks: 'How many are these of your numerous herd?' The shepherd answers: 'These are $\frac{2}{3}$ of $\frac{1}{3}$ of my cattle; calculate their number for yourself.'"

2. From a Chinese arithmetic: "In a stable there are pheasants and rabbits: They have together 35 heads and 98 feet; how many animals of each kind are there?"

3. A donkey asked a horse whether his load was heavy; the horse replied, "My load is thrice as heavy as yours, but if I had yours and mine together it would be only half a ton; find the result yourself."

4. The sum of the present ages of Mary and Ann is 30 years; six years ago Mary was twice as old as Ann is now; what are the present ages of Ann and Mary?

5. A boy being asked his age replied, "Twice my age 4 years ago was the same as my age 4 years hence"; what is his present age?

6. A hare is 300 yards ahead of a hound; the hound runs 3 times as fast as the hare; how far does the hare still run before it is caught, if the hound overtakes it in 1 min.?

7. A snail crawling up the side of a well 30 ft. deep slips back each night $\frac{1}{2}$ as far as it crawled during the day; it gets out in 9 days, not slipping back after the last day; how far does it crawl up each day?

8. A jewel casket and lid weigh 27 oz.; the lid is $\frac{2}{7}$ as heavy as the casket; what is the weight of the lid?

9. From a medieval arithmetic: "A master bargained with a servant to give him 10 gulden (pieces of money) a year and a coat. The servant remained only 7 mo. The master said, 'Leave my house and take the coat that I gave you; I owe you nothing more.' How many gulden was the coat worth?"

Oral.

1. When x dollars buys 10 yd. of silk, what does 1 yd. cost? $\frac{1}{3}$ of a yard? $\frac{2}{3}$ of a yard?

2. When c cents buys 12 oranges, what does 1 orange cost? If d dozen oranges cost a dollars, what is the cost of 1 orange?

3. If a man can do a piece of work in n days, how much of it can he do in 1 day? In 4 days? In d days? How much of it could 3 men do in a days? In x days? How much could q men do in 1 day? In t days?

4. There are a rows of b trees each in an orchard; how many trees are there?

5. A rectangular pile of bricks is a bricks long, b wide, and c high; how many bricks are there in the pile?

6. Represent the length of a rectangle by l in. Its breadth is 12 in. less than its length; what represents its perimeter?

7. State and illustrate some processes which may be performed on both members of an equation without destroying the equality.

8. Clara paid c cents for ribbon and twice as much for silk, spending in all $48¢$; how much did she pay for each article?

Written.

9. One box weighed p pounds, and another weighed 10 lb. more than the first. The two together weighed 130 lb.; what did each weigh?

10. A man 48 years of age is 3 years more than 5 times as old as his youngest child; how old is the latter?

11. If Mr. Brown's annual salary were increased by $\frac{1}{5}$ it would lack \$600 of being \$1,800; what is his salary?

12. $\frac{1}{3}$ of a certain sum is invested at 6%, the remainder at $4\frac{1}{2}\%$. The annual income is \$45; what is the capital?

Written.

1. The sum of two numbers is 384; one of them is 31 greater than the other. Find the numbers.

2. A freight train consisted of 60 cars; the number of closed cars was 6 more than twice the number of open cars. Find the number of each.

3. At a certain election there were two candidates; 1,147 votes were cast, and the candidate chosen had a majority of 259; how many votes did he receive?

4. A library contains 1,000 books; a certain number were drawn out on Monday and twice that number on Tuesday; 880 books were left; how many books were drawn on Monday?

5. A society sent two delegates to a convention. To defray their expenses ($34), the members paid $.75 each into the treasury and thus contributed $8 more than the amount required; how many members were there in the society?

6. The distance from Chicago to Des Moines, Iowa, is 374 miles. If a train runs this in $8\frac{1}{2}$ hr., what is its speed per hour? How far would this train run in h hours? How many hours would be required to run 100 miles? To run m miles?

7. Of a company of soldiers, $\frac{1}{3}$ are detached to build breast-works; 15 others are detached for scout duty; $\frac{1}{2}$ of the whole company still remain in the firing line; how many are there in the company?

8. Find a number such that the quotient when it is divided by 3 is 8 more than when it is divided by 6.

9. Two persons have the same annual salary. The first saves $\frac{1}{6}$ of his income. The second, who saves $350 a year more than the first, has $3,000 at the end of 4 years. Find the salary.

XVI

INDUSTRIAL APPLICATIONS

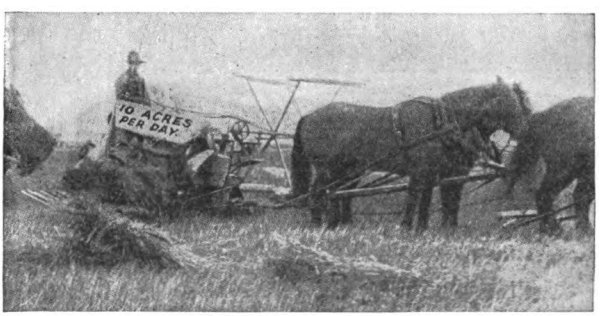
THE WORLD'S BREAD SUPPLY—WHEAT

Oral.

1. The large wheat farms of Minnesota and the Dakotas contain from 3,000 to 10,000 acres of land. Using $1\frac{1}{2}$ bu. of seed wheat an acre, how many bushels of seed would be used to sow 3,000 acres? 10,000 acres?

2. How many bushels of wheat would a farmer harvest from 3,000 acres yielding 15 bu. an acre?

3. What would this wheat bring at 60¢ a bushel?



Written.

4. In a field of 3,000 acres 20 reapers were used, each having the daily capacity indicated in the picture; how many days did it take to cut the grain?

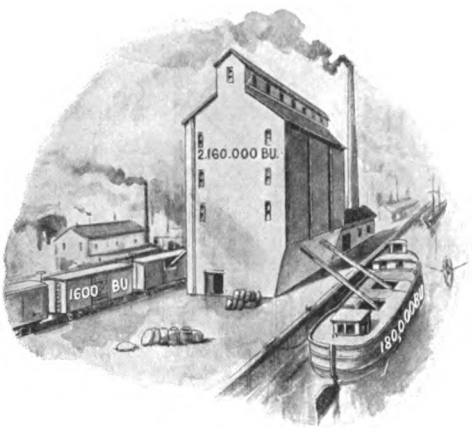
5. The grain was threshed in the field by 10 threshing machines, each having a capacity of 400 bu. a day; how many days did it take to thresh the wheat? See Exc. **2.**

6. The grain was loaded on cars carrying 900 bu. each; how many cars were required for the crop?

7. $4\frac{1}{2}$ bu. of wheat are used to make a barrel of flour; how many barrels would this crop make?

Oral.

Wheat is sometimes shipped directly from the fields to consumers, but it is usually stored in buildings called *elevators* to be reshipped later.



1. How many boatloads of 180,000 bu. each does the elevator in the picture hold? How many carloads?

2. At $\frac{1}{2}¢$ per bushel, what is the cost of filling the elevator once?

Written.

3. How many times would the annual wheat yield of Minnesota (about 80,000,000 bu.) fill the elevator shown?

4. Recently, Minnesota's wheat crop (see Exercise 3) was $1\frac{1}{3}$ times as large as that of Great Britain, twice that of Australasia, and $6\frac{2}{3}$ times that of Egypt. How many bushels were grown by each?

5. The total wheat crop of the United States in a certain year was 670,000,000 bu., of which 40% was consumed at home; how many bushels were exported?

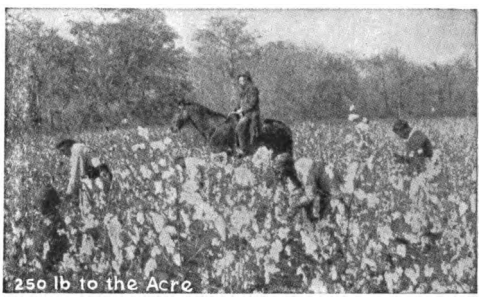
6. Recently, America produced 769,000,000 bu., or about $\frac{1}{4}$ of the world's production; what was the approximate production of wheat in that year? Europe produced $\frac{1}{2}$ of this amount; how many bushels was this?

7. There were in the same year 550,000,000 bread eaters in the world, consuming $4\frac{1}{2}$ bu. per capita; how many bushels were consumed? Was the wheat crop sufficient to meet the demand?

Oral.

1. The lower Mississippi basin and Atlantic slope is the greatest cotton producing region in the world. Its area is $\frac{1}{6}$ of that of the United States. Taking the latter to be 3,000,000 sq. mi., how many sq. mi. are there in the cotton region?

2. The cotton plantation shown in the picture contains 600 acres; according to the yield given, how many pounds of cotton does it produce in a season?



3. Cotton is pressed into bales of 500 lb. each, if of full weight. How many acres of this plantation are required to grow 1 bale?

4. How many bales are produced by the plantation?

5. The planter sells his cotton for 10¢ a pound; find the value of one season's product from this plantation.

Written.

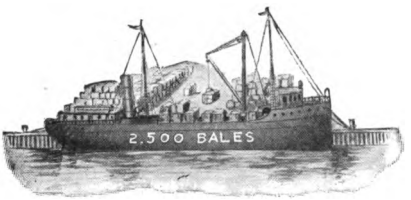
6. 28 million acres of cotton were grown in the United States in 1903. The total crop was 10,758,000 bales. What was the average yield in bales per acre? In pounds per acre?

7. What was the total value of this product at 12¢ a pound?

8. Nearly all of the raw cotton grown in the United States is shipped to the northern states or to Europe, for manufacture. 6,482,849 bales were exported to Europe in 1903. According to Exercise **6**, how many bales were retained at home? What per cent (within 1%)?

Written.

1. How many tons will the cargo of this ship weigh at 500 lb. per bale?

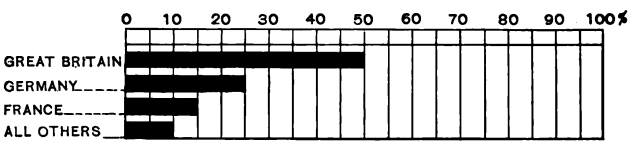


2. What is its value at 11¢ a pound?

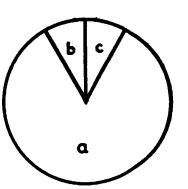
3. How many such cargoes would it require to transport the 6,482,849 bales of cotton exported to Europe in 1903?

4. Texas is the largest cotton producing state in the United States. In 1903 the crop was 2,575,000 bales; what percentage was this of 10,758,000 bales, the total production in the United States (within 1%)?

5. The diagram shows how the cotton exported in one year was distributed; what percentage was shipped to Great Britain? To each of the other places?



6. The world's supply for the same year was produced as follows: America produced 10 million bales, the West Indies 1 million, all others 1 million bales; name the parts of the circle which represent each of these quantities.

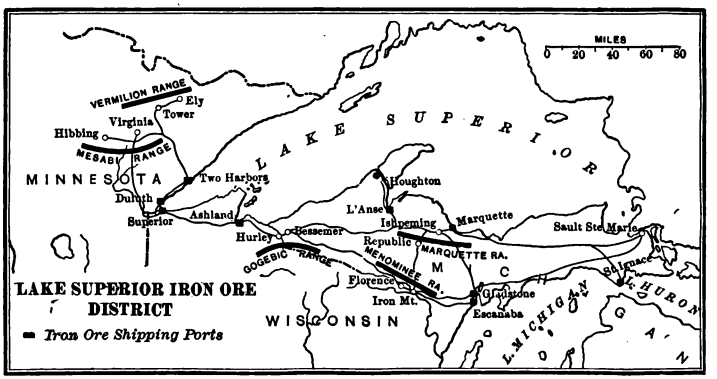


7. The average weight was 485.3 lb. a bale; how many tons were there in the world's supply?

8. Indicate graphically that all of the spindles spinning the cotton were divided thus: Great Britain, 47 million; the European Continent, 34 million; the United States, 22 million; and the East Indies, 5 million.

Oral.

1. What states border on Lake Superior? How many iron ranges are shown in the map?



2. The production has reached 12,000,000 tons in one year; what was the average production for each range?

3. Much of the ore is taken with steam shovels, averaging 1,800 tons a day; how many days would it take 25 such shovels to load 8,000,000 tons of ore?

4. What was the cost of mining this output at 25¢ a ton?

5. The other 4,000,000 tons were mined underground; what was the cost of mining at 90¢ a ton?

Written.

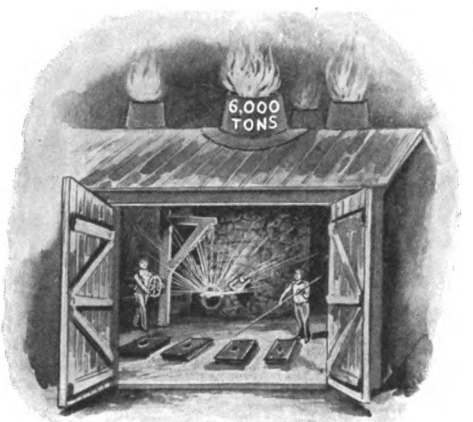
6. 3,000,000 tons have been shipped in one year from the ports of Lake Michigan and Lake Superior. Count the iron ports on the map and find the average tonnage per port.

7. The iron and steel produced in the United States in a recent year was 33 million tons. This was about $\frac{11}{27}$ of the amount produced in all countries; how many million tons were there in the world's production?

8. The amount of iron used by the people of the United States averages 300 lb. per capita; what was the consumption in 1900 when the population was 76 million?

Written.

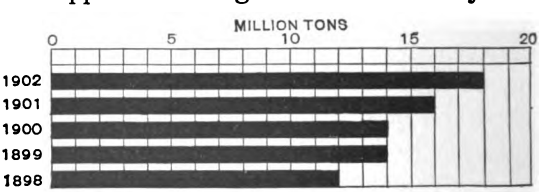
1. Iron ore is melted in blast furnaces with coke and limestone and drawn off into bars called pig iron. A ton of coke and $\frac{1}{2}$ of a ton of limestone are required for every $1\frac{2}{3}$ tons of ore; how many tons of coke and how many tons of limestone are required to reduce 3,000,000 tons of ore?



2. The cut shows the daily capacity of a modern furnace; how many tons does such a furnace produce in 365 days?

3. 12 new blast furnaces, averaging 5,000 tons daily in capacity, were recently built in one year. How many tons of pig iron do these furnaces produce annually?

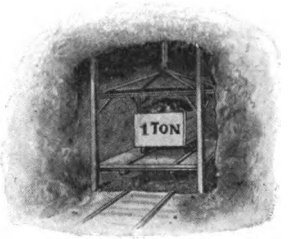
4. About $\frac{1}{3}$ of the pig iron produced in the United States is produced in the Appalachian region between Maryland and Alabama. From the diagram read the total product of the United States, and find the number of tons produced in the Appalachian region in each year named.



5. Represent graphically the world's supply of pig iron in 1902: United States, 18 million; United Kingdom, 8.6 million; Sweden, .5; Spain, .3; Russia, 2.5; Germany, 8.4; France, 2.4; Canada, .3; Belgium, 1.1; Austria, 1.3.

Coal is usually mined by sinking deep shafts into the earth until the coal deposits are reached. The coal is broken up by blasting, and raised to the surface by elevators or run out of horizontal tunnels on cars.

Written.



1. How many cars of coal like that shown do 100 miners produce in a day, each averaging 5 tons a day?

2. How much do they mine in 4 months of 26 days each?

3. What does it cost to mine the coal mentioned in Exercise 2 at 50¢ a ton?

4. The area of the United States is 3,029,600 sq. mi. The coal fields underlie $\frac{1}{16}$ of the country; how many square miles is this?

5. In a recent year, 258,372 tons of bituminous or soft coal were mined in the United States, taken from the following regions. How many tons were produced by each?

Appalachian field.....	65%	Western field.....	12%
Central field.....	15%	Other fields.....	8%

6. How many 50-ton carloads would the total product make?

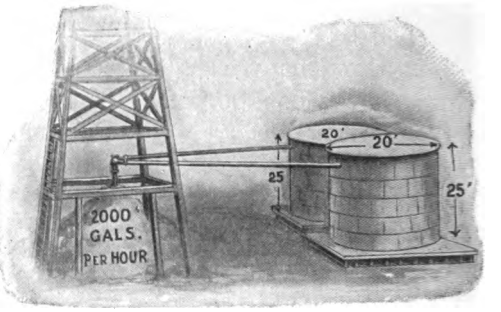
7. Represent graphically the coal production of the world in a recent year:

COUNTRY.	MILLION TONS.	COUNTRY.	MILLION TONS.
United States..	266	France.....	32
British Isles....	223	Belgium.....	22
Germany.....	153	Russia.....	16
Austria.....	40	All others.....	25

8. Taking the total of Exercise 7 to represent the world's annual consumption and 650,000,000,000 to be the total deposits; how many years will it take to exhaust the supply?

Petroleum is crude oil found in pockets below the surface of the earth. Usually the oil flows freely when an iron pipe is sunk to one of these pockets. It flows at various rates—in some cases 3,000 gallons an hour.

Written.



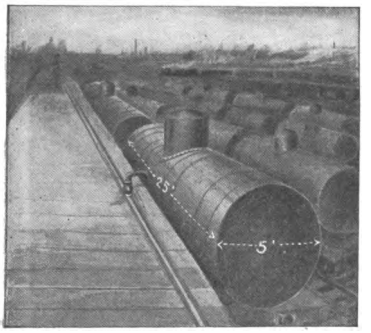
1. How many gallons does a well flowing at the rate mentioned above produce in a day? In a week? In a year?

2. Answer the same questions for the well shown in the picture. How many gallons do the two tanks hold? ($7\frac{1}{2}$ gal. = 1 cu. ft.)

3. How long would it take this well to fill both tanks?

4. A pipe-line for carrying oil from Pittsburg to New York City delivers 300 gal. a minute; how many gallons does it deliver in a day? In 365 days?

5. The capacity of a tank ship may be taken as 400,000 gal.; how many such cargoes does the pipe-line mentioned deliver in a year?



6. At one time the pipe-lines from the McDonald field in Pennsylvania had a daily capacity of 5,400,000 gal.; how many cargoes of oil did they deliver in 30 days?

7. The refined oil is shipped in tank cars. How many gallons does this car carry? What is the value of such a carload of kerosene at 9¢ a gallon? Of gasoline at 13¢ a gallon?

1. Taking the population of the United States to be 76,000,000, how many bushels of wheat are required to feed the people, allowing $4\frac{2}{5}$ bu. to each inhabitant?

2. The wheat crop of the world in a recent year was 2,226,745,000 bu. This was only 95% of the demand; how many bushels did the crop lack of supplying the demand?

3. In another year the crop was 2,879,924,000 bu., which was 113% of that year's demand; how many bushels were there in excess of the demand?

4. 47,000,000 acres of wheat were sown in this country in one year; how many bushels of seed were required, allowing $1\frac{1}{2}$ bu. to the acre?

5. 50,000 men are engaged in the Dakotas during a wheat harvest. At $65 a month how many dollars does it take to pay them for July and August?

6. In a recent year Minnesota sowed 5,000,000 acres of wheat and harvested 78,000,000 bu.; what was the yield per acre?

7. Pennsylvania consumed $\frac{9}{13}$ of a wheat crop of 26,000,000 bu.; how many bushels were shipped out of the state?

8. A cargo of wheat on the Great Lakes contained 360 carloads of 700 bu. each; how many bushels was this?

9. A New England cotton mill uses 15 bales of 500 lb. each a day; how many tons does it use in 300 days?

10. This mill employs 600 hands and has a weekly pay roll of $2,000; what is the average weekly wage for each employee?

11. A cotton plantation of 600 acres yields 250 lb. of cotton an acre; how many 500-pound bales are produced? What is the value of the cotton at 12¢ a pound?

12. For each pound of cotton of Exercise 11 $2\frac{1}{2}$ lb. of seed were obtained; how many pounds of cotton seed did the plantation yield?

1. Refined oil is exported in 5-gallon cans. A certain factory for making cans has a daily capacity of 70,000 cans; how many gallons of oil can be shipped in the cans that it makes in one year (300 days)?

2. In 1880 two men could solder 1,000 cans in a day. By use of the latest machinery, three men can now solder 24,000 cans in a day; taking the wages to be the same, the cost of soldering a can now is what part of the cost in 1880?

3. In a recent year the Standard Oil Company imported 60,000 tons of tin, which cost $1,000,000; how much was this a pound?

4. In 1872 the United States exported 16,363,975 gal. of crude oil, in 1903 it exported 134,892,120 gal.; what was the percentage of increase?

5. The world's production of petroleum in 1904 was 5,000,000,000 gallons, of which the United States produced $2\frac{1}{2}$ billions and Russia $2\frac{1}{4}$ billions; how many gallons were produced by all other countries?

6. In 1880 there was built on an average 1 oil tank a day. The tanks averaged in capacity 30,000 barrels; what was the total tankage added in this year of 366 days?

7. The average cost of the tanks mentioned in Exercise 6 was $8,500 each; what did the tanks built in 1880 cost?

8. What is the value of 1,500 tons of steel rails at $28 a ton?

9. The following list shows the annual capacities of the leading steel manufactories; find the total capacity:

Carnegie.....	2,000,000 tons	National	2,580,000 tons
Federal Steel...	2,310,000 "	Am. Steel and Wire	935,000 "

10. In 1890 the value of the steel exports of this country was $27,000,000. Their value in 1900 had increased 480%; what was this value? In 1897 the value was $62,737,000; what was the percentage of increase from 1897 to 1900?

XVII

COMMERCIAL APPLICATIONS

ORDERING GOODS

1. Make out a bill for the following order for goods, the price of the bed spring being $4.50.

18 Main Street, Rockwood, N. J.

June 8, 1905.

Mr. John Wanamaker,

New York City.

Dear Sir:

Please send me by express:
1 set Dickens' complete works, advertised at $18,
1 3-foot woven wire bed spring, Cat. No. 872,
1 Sable fur boa, advertised at $7.95,
and charge to my account.

Yours truly,

John Perkins

Write an order and a receipted bill for each of the following; the purchases being made of Montgomery Ward & Co., Chicago:

2. Roland Ames, Springfield, Missouri:

2 lawn mowers, catalogue number 6900, each $2.33.
2 doz. camp chairs, " " 7053, " .70.
½ doz. folding cots, " " 7770, " 1.65.

3. A. R. Childs, Newark, New Jersey:

Two 5-pound cans Java coffee, at 31½¢ a pound.
5 bbl. flour, catalogue number A 723, at $5.50 a barrel.
3 doz. cans sardines, catalogue No. A 271, at $4.25 a doz.

4. Walter Fields, Memphis, Tennessee:

3 chairs, catalogue number R 81, at $10.50 each.
1 Davenport sofa, catalogue number R 97, at $45.00.

Day Book. Business houses vary greatly in their methods of keeping account of transactions. A common method of recording sales is that of noting in a book, called a *day book*, the name of the customer, a description of the goods bought, and whether the price is charged to the customer's account or paid in cash.

The following is a coal dealer's record of one day's sales:

February 18, 1905.		Account		Cash	
Thomas Carlton	½ Cord wood			1	25
Ray Rankin	Nut Coal 300 lb.				88
Arthur Ailey	Smith . 170 "				50
James O'Brien	Stove . 80 "				25
Matthew Spencer	Nut . 2142 .			6	17
M. C. Carr	Steam . 4000 .	8	00		
Harold Jobson	Nut . 80 .				25
Peter Mason	Nut . 8000 . .	22	00		
H. A. Goodrich	Nut . 625 .			1	64
Myron Cobb	Steam . 2600 .	5	93		
Adam Case	Nut . 4000 .			12	00
Herbert Nixon	½ Cord Wood	1	75		
	Totals	39	18	33	94

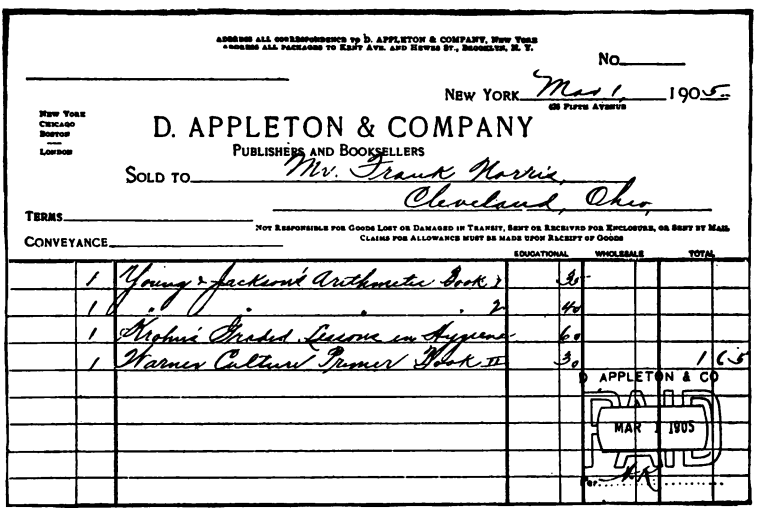
Written.

1. Verify the totals in the record above, and find the total amount of the sales for the day.

2. On a certain day a dealer's cash items were: $2.50, $5.75, $11.16, $4.93, $.30, $1.15, $.33, $4.60; the account items were: $4.78, $13.60, $5.12, $3.17; find the total.

3. Make other records and find the totals.

Receipts. When a bill is paid it is customary to give a written acknowledgment of payment, called a *receipt*. The receipt may either be written or stamped on the bill itself, as in the bill shown below, or drawn up separately.



ADDRESS ALL CORRESPONDENCE TO D. APPLETON & COMPANY, NEW YORK
PACKAGES ALL PACKAGES TO EIGHT AVE. AND HOUSTON ST., BROOKLYN, N. Y.

No.

NEW YORK May 1, 1905
436 FIFTH AVENUE

NEW YORK
CHICAGO
BOSTON
LONDON

D. APPLETON & COMPANY
PUBLISHERS AND BOOKSELLERS

SOLD TO Mr. Frank Harris
Cleveland, Ohio

TERMS

NOT RESPONSIBLE FOR GOODS LOST OR DAMAGED IN TRANSIT, SENT OR RECEIVED FOR ENCLOSURE, OR SENT BY MAIL.
CLAIMS FOR ALLOWANCE MUST BE MADE UPON RECEIPT OF GOODS

CONVEYANCE

			EDUCATIONAL	WHOLESALE	TOTAL
	1	Young's Jackson's Arithmetic Book I	36		
	1	" " " " 2	48		
	1	Hyde's Graded Lessons in Hygiene	60		
	1	Warner Coltrin Penmanship Book II	30		1 65

D. APPLETON & CO
MAR 1 1905
per

Make out receipted bills for the following items, with yourself as purchaser and as seller some firm that you know deals in the commodities named:

1. 3 sacks of flour at $1.69; one 16-pound ham at 12½¢ a pound; 9 lb. of tea at 39¢ a pound.

2. 1 dining table, $27; 8 chairs at $2.98 each.

3. 6 violin strings at 19¢ each; 1 cake rosin, 8¢; 3 copies of sheet music at 29¢ each, 5 at 15¢, and 1 at 60¢.

Write receipts showing that you have made the following payments:

4. A monthly instalment of $10 for the current month on a piano that you have bought from a dealer known to you.

5. To the publisher of a local paper or magazine for one year's subscription in advance. (Use correct name and rate.)

Cash Accounts. An account of money received and paid out is called a *cash account.*

Dr. stands for debtor and *Cr.* for creditor. "Cash" may be thought of as a debtor to the person keeping the account for all money received and creditor for all money paid out.

Cash Account of M. R. C.

			Dr.		Cr.	
Sept.	1	Cash on hand	16	15		
"	1	Wages	13	50		
"	2	Rent			14	00
"	3	Hat			1	50
"	5	School Books			3	62
"	6	Groceries			5	07
"	7	Milk Tickets			1	00
		Balance			4	26
			29	65	29	65
Sept.	8	Cash on hand	4	26		

Written.

Make out and balance the following accounts:

1. Cash on hand Jan. 1, $500. Expenditures: Jan. 5, $80.30; Jan. 10, $30.75; Jan. 15, $4.25; Jan. 21, $106; Jan. 30, $40.80.

2. Cash on hand Monday, $90.50. Receipts: $10 a day for the week of 6 days. Expenditures: Monday, $7.50; Tuesday, $12.25; Thursday, $18.40; Saturday, $21.43.

3. Cash on hand July 1, $74.90. Receipts: July 6, $60.70; July 15, $50; July 21, $18.90; July 31, $50. Expenditures: July 2, $25.50; July 7, $6.50; July 16, $40.59.

4. Cash on hand Oct. 1, $185.50. Expenditures: Oct. 3, $16.50; Oct. 11, $89.70; Oct. 15, $5.53; Oct. 25, $17.75; Oct. 28, $15.90; Oct. 30, $12.

5. Make and solve 5 similar problems.

Sending Money by Mail. Money may be forwarded by mail in a letter which is **registered** at a post-office. The fee for this is 8 cents. A receipt showing the delivery of the letter is returned to the sender by the post-office.

Sending Money by Postal Money Order. Payment to a person at a distance may also be made by sending him a *postal money order* purchased at a post-office. This is an order directing the postmaster at the place where the other person lives to pay him the money.

The fees charged for money orders are:

For orders for sums not exceeding						$2.50	3 cents
"	"	over $2.50	and	not	exceeding	5.00	5 "
"	"	" 5.00	"	"		10.00	8 "
"	"	" 10.00	"	"		20.00	10 "
"	"	" 20.00	"	"		30.00	12 "
"	"	" 30.00	"	"		40.00	15 "
"	"	" 40.00	"	"		50.00	18 "
"	"	" 50.00	"	"		60.00	20 "
"	"	" 60.00	"	"		75.00	25 "
"	"	" 75.00	"	"		100.00	30 "

The money is not actually sent from one post-office to the other. The transfer is merely a matter of book-keeping, since the government has received the money at one office and paid it out at another.

Sending Money by Express. Money may be shipped by express, and large payments in gold are so shipped. But small amounts are usually paid by an *express money order*.

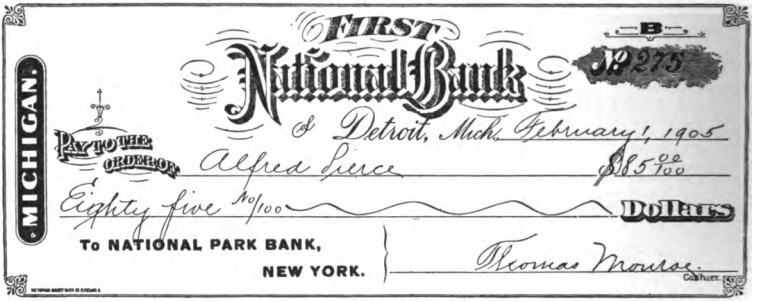
The plan and rates are similar to those of the postal order.

Sending Money by Check. A check, drawn upon a bank in which the sender has a deposit, may also be sent. The person receiving the check will present it at a bank in his locality. That bank will collect the check from the bank on which it was drawn.

Many banks charge a small fee for collecting a check, and hence it is usually preferable to use some other form of sending money.

NOTE.—Fuller treatment of money orders, with problems, may be found in Book II, pages 40 and 41. Checks are treated in Book II, p. 208.

Bank Drafts. Since a charge for collection is usually made only upon personal checks, it may be avoided by procuring a *bank draft*. This is merely a check drawn by one bank upon another, and may have the form here shown.



Mr. Pierce will indorse this draft in full and mail it to Mr. Prince, to whom he wishes the money paid. The National Park Bank at New York will pay Mr. Prince $85 on presentation of the draft. The indorsement is as here shown.

Pay to the order of
H. O. Prince.
Alfred Pierce.

1. Write a draft drawn by the cashier of the Mechanics' Bank of Rochester for yourself payable at the National Park Bank of New York City.

2. Indorse this draft in favor of James Wanamaker.

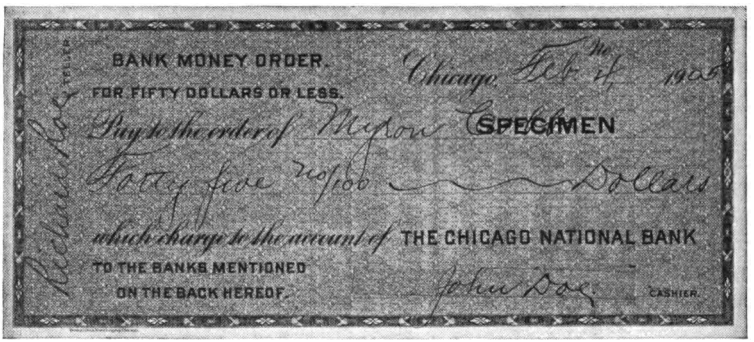
3. The sender may have the draft drawn payable to the creditor, in which case no indorsement is necessary. Write such a draft.

Oral.

4. Why do business houses prefer bank drafts or money orders to personal checks?

5. Name four ways of making remittances; what one is the cheapest to the sender?

Bank Money Orders. Banks often issue money orders. The fees vary. The following is a specimen order:



On the back of such orders a list of the banks is printed which have agreed to pay these orders when presented. But usually any bank will pay them on satisfactory identification of the holder. The orders are marked "Fifty dollars or less," "One hundred dollars or less," "One thousand dollars or less," etc., according to the amount of the order.

Premium. The amount charged by a bank for an order is called *premium*.

Oral.

1. What is the premium at $\frac{1}{4}\%$ on an order for \$1,000? What is the total cost of such an order?

2. What is the total cost of an order for \$2,500 at $\frac{1}{8}\%$ premium?

3. Name two other kinds of money orders.

4. Name five ways of remitting money.

Written.

5. What is the total cost of orders for the following sums, premium $\frac{1}{4}\%$:

\$1,500	\$3,500	\$10,650	\$1,225
1,625	86.50	195.50	40.30

Sending Money by Telegraphic Order. The most rapid method of remitting money is by telegraph. The sum to be remitted is paid to the agent of a telegraph or express company who telegraphs the agent at any other place to pay out an equal sum to the person named. The money is at once delivered by messenger.

The principle is the same as that of postal money orders, but the fee is higher. A common charge is 1% of the sum sent in addition to double the usual rate for a 10-word message. The following are the rates for such messages from Chicago to the places named:

San Francisco	75 cents	New Orleans	50 cents
Detroit	25 "	Galveston	60 "
New York	40 "	Kansas City	30 "

Oral.

1. What is the cost of sending $150 by telegraphic order from Chicago to San Francisco, at the charge above?

2. Make and solve 5 similar problems.

Cabling Money. The cost of cable messages is often reduced by using one word to represent several. A list of such words is called a **code.** Both the sender and the receiver must have the code. The following will illustrate:

WORD.	MEANING.
Fichant	One hundred dollars.
Feststadt	Fifty dollars.
Ficheron	One thousand dollars.
Cicatrix	American Express Company's office.
Fenderent	Have cabled $— through —— at ——.

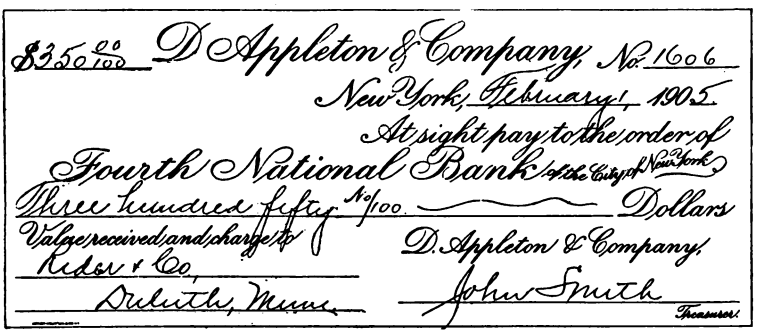
Mr. Marks, traveling in London, receives the following message from Mr. Case in New York City.

Marks, Ten Oxford Street, London.
Fenderent Ficheron Cicatrix London. Case.

3. Translate the above message. What did it cost at 25¢ a word, including the address and signature? What was the cost of cabling the money at $1\frac{1}{2}\%$ of the sum sent?

4. Make and solve other problems.

The firm, Rider & Company of Duluth, owe D. Appleton & Company of New York $350 for books. D. Appleton & Company may collect the account by means of the following draft:


$350 00/100 D. Appleton & Company No. 1606
New York, February 1, 1905.
At sight pay to the order of
Fourth National Bank
Three hundred fifty 00/100 Dollars
Value received and charge to
Rider & Co.
Duluth, Minn.
D. Appleton & Company.
John Smith
Treasurer.

This is sent by the Fourth National Bank of New York to a bank of Duluth. This bank sends the draft by messenger to Rider & Co., who may accept the draft and pay the amount. If they refuse to pay it, the draft is returned to its maker.

Kinds of Drafts. Such a draft is called a **commercial draft,** and, since it was payable at sight, a **sight draft.** Instead of the words "at sight," the drawer may state any future time of payment. In this case, the draft is called a **time draft.** When the draft is presented, the debtor writes "accepted" with date and signature across its face, if he intends to pay it. An accepted time draft may be discounted like a note. See page 86.

1. The bank making the collection may charge a fee; what would be the cost of collecting this draft at $\frac{1}{2}\%$?

2. A time draft for $325 had been accepted; the holder discounted it 30 da. before it was due at 6%; what were the proceeds?

3. What is the cost of collecting a draft for $4,500 at $\frac{1}{4}\%$?

4. A time draft for $1,500 had been accepted; the holder discounted it 2 mo. before it was due at 6%; what were the proceeds?

Brokers. The commission merchant is no longer the chief factor in marketing the important crops of the country, as wheat, corn, and cotton. Agents of commercial houses buy the grain directly from the farmers and ship it to the great cities for immediate sale or to be stored in elevators. Samples graded by an official inspector are taken to a place called the "Chamber of Commerce" or "Board of Trade," where agents called *brokers* negotiate sales of these commodities.

Brokerage. The fee paid the broker for buying or selling a commodity is called *brokerage.* It is usually a fixed sum per bushel, barrel, bale, etc., irrespective of the price of the commodity.

Rates of Brokerage. Some common rates of brokerage are:

For grain, $\frac{1}{8}¢$ a bu., the least charge being $2.00.
For cotton, 5¢ a bale, the least charge being $5.00.
For pork, $2\frac{1}{2}¢$ a barrel, the least charge being $6.25.
For coffee, 4¢ a bag of 130 lb.

Oral.

Find the brokerage on each of the following sales:

1. 10,000 bu. wheat.
2. 2,000 bales cotton.
3. 100 barrels pork.
4. 500 bags coffee.
5. 210,000 bu. wheat.
6. 1,000 barrels pork.
7. 1,000 bu. corn.
8. 5,000 bu. oats.
9. Make and solve 10 similar problems.

10. An elevator firm sold "10 wheat" (10,000 bu.) through a broker at 65¢ a bu.; what was the brokerage? How much did the firm realize from the sale?

11. A firm bought 3,000 bales of cotton on the Exchange at $50 a bale; how much did it cost?

12. A broker bought on his own account 20,000 bu. of wheat at 65¢ a bu. and sold it at 78¢; how much did he gain?

Fire Insurance. When a company insures a house for $2,000 against loss by fire, it agrees to make good the owner's loss to the extent of $2,000, if the house is accidentally burned during the period specified.

Premium. In return, the owner pays the company in advance a stipulated amount called a *premium.*

Policy. The contract between the company and the owner is called a *policy.*

Face. The amount for which the property is insured is called the *face* of the policy.

1. A house is insured against fire for $3,000; the rate is $.65 per $100 per annum; what is the annual premium? Two annual premiums in advance usually pay for 3 years' insurance; what is the amount paid in advance for three years' insurance on the house?

2. Mrs. Brown had her jewels and wardrobe insured against fire for $3,000; the rate was $.45 per $100 per annum; what was the annual premium? Three annual premiums in advance usually pay for five years' insurance; what did Mrs. Brown pay for five years' insurance?

Accident Insurance. In accident insurance, the company agrees to pay the person insured a certain sum per week or month, as long as he is incapacitated through injury by accident. If he is permanently maimed, stipulated sums are paid, and if he dies in consequence of an accident, a certain sum is paid to a beneficiary named in the policy.

3. A mechanic paid $25 a year for 5 years for an accident policy which provided these benefits: $25 a week for total loss of time through accident; $10 a week for partial loss of time. He was totally disabled 8 weeks and partially disabled 6 weeks. How much did he receive in benefits?

Other Insurance. Insurance against loss by tornado, lightning, theft, disaster at sea (marine insurance), etc., is based on the same principles as fire insurance.

Life Insurance. The essential feature of life insurance is, that the insurance company pays a fixed sum to a person named (**beneficiary**), whenever the insured person dies. In many cases the condition is added that the sum shall be paid to the insured himself, if living at the end of a stipulated time; for example, 20 years. The first may be called a **life policy,** the second, **a 20-year policy.**

Premium. In return the insured person pays (usually annually in advance) a fixed sum called a *premium.*

The amount of the annual premium depends upon the age of the insured at the time when the payments are begun, and the character of the policy. It is usually stated at a rate per $1,000 of the insurance. There are many different kinds of policies, with different conditions and privileges. The following specimen table of rates may be used in the problems:

Age when policy is issued..	21	22	23	24	25
Annual premium, life.....	15.01	15.35	15.71	16.03	16.46
" " 20 yr. ...	42.44	42.50	42.55	42.61	42.66
Age when policy is issued..	26	27	28	29	30
Annual premium, life.....	16.37	17.31	17.76	18.24	18.74
" " 20 yr. ...	42.73	42.80	42.88	42.97	43.07
Age when policy is issued..	31	32	33	34	35
Annual premium, life.....	19.27	19.83	20.42	21.04	21.70
" " 20 yr. ...	43.17	43.29	43.41	43.67	43.73

1. Mr. Gray insured his life for $1,000 at the age of 21 on the 20-year plan. At the expiration of the twenty years he was still living and received $1,000; how much more did he receive than he had paid in?

2. How much does a man beginning at 35 years of age pay for $1,000 of insurance on the life plan, if he lives to be $75\frac{1}{2}$ years of age?

3. Make and solve five problems about life insurance.

1. A man deposited $2,000 in a savings bank, July 1, at 3% per annum; what interest was credited the following Jan. 1? What was the new principal?

2. What was the interest and new principal the next July 1? How much money would the man have in the bank at the end of 3 years?

Simple Interest. Interest which is computed on the original principal only is called *simple interest.*

Compound Interest. When interest due is added to the principal, and interest is paid on the whole thereafter, the total interest so paid is called *compound interest.*

When the interest is added to the principal semiannually, it is said to be *compounded semiannually:* when added annually, it is said to be *compounded annually.* Compound interest is almost invariably computed by the use of tables.

AMOUNT OF $1 AT COMPOUND INTEREST, COMPOUNDED ANNUALLY: —

YEARS.	RATE: 2%	2½%	3%	4%	5%
1	1.02000	1.02500	1.03000	1.04000	1.05000
2	1.04040	1.05063	1.06090	1.08160	1.10250
3	1.06121	1.07689	1.09273	1.12486	1.15763
4	1.08243	1.10381	1.12551	1.16986	1.21551
5	1.10408	1.13141	1.15927	1.21665	1.27628

Thus, $1 at 3% compounded for 4 years amounts to $1.12551. This is found in the column under 3% and in the line opposite 4 at the left. The compound interest on $100 for 4 years at 3% would be 100 × $1.12551, and similarly for any other amount.

Interest otherwise compounded can easily be found from it.

Thus, interest at 4% per annum compounded semiannually is the same as interest at 2% compounded annually for twice the number of years.

Find from the table the amounts of the following, compounded annually:

	PRINCIPAL.	TIME.	RATE.
3.	$1,000	5 years	3%
4.	500	3 "	4%
5.	650	4 "	2%

Oral.

1. Mr. Carelton of Detroit owes Rice and Co. of St. Louis $175 for some goods; how may he discharge the indebtedness? Will he be likely to use an express or postal order or bank draft?

2. What is a bank draft? What is a commercial draft?

3. What is a sight draft? A time draft? How is a commercial draft collected?

4. Smith & Co. of New York draw upon Jones & Co. of Johnstown for $250, 30 days after sight; what may Jones & Co. write on the face of the draft to show that they will pay it?

5. What is the usual rate of brokerage for buying and selling grain on the Exchanges? At this rate find the brokerage on the following purchases:

32,000 bu. of wheat.	10,000 bu. of barley.
20,000 bu. of oats.	7,000 bu. of corn.

6. A firm bought 5,000 bales of cotton at $50 a bale, brokerage 5¢ a bale; what did it cost?

Written.

7. Write an order to Sargent and Smith, 18 Grand St., New York, for a tool chest which they advertise in The Youth's Companion for $8.95, purchaser to pay carriage. State in what form you send the money and give directions as to sending the tool chest to you. When ready for shipment it weighs 65 lb. (If convenient, ascertain which would be the cheaper way of having it come to you, by freight or by express, remembering that you must also pay cartage from the freight depot to your home, if sent by freight.)

8. Make and receipt a bill for 5 tons of hard coal at $6.75; carrying in, 25¢ per ton; 1 load of kindling wood, $1.25.

XVIII

STOCKS AND BONDS

STOCK COMPANIES

Companies. When several persons wish to conduct a business enterprise together, they organize a *company*.

Capital. The money invested in business is called *capital*.

Corporations. If the organization of the company is recognized by the state (in accordance with certain laws) the company is said to be **incorporated.** When incorporated, the company (*corporation*) can legally transact business like an individual.

Stock. The capital of a corporation is divided into equal amounts called **shares.** In this case the capital is called *capital stock*, or simply *stock*. Whatever amount a share represents is called the **face value,** or **par value,** of the share.

In what follows, the par value of a share is to be regarded as $100, unless otherwise specified.

Capital is invested in a stock company by the purchase of one or more of the shares of stock.

Stock Certificates. Certificates are issued for the shares of stock. The owners of stock are called **stockholders.** A certificate of stock is shown on page 170.

Dividends. The business is carried on under the management of officials elected by the stockholders. At regular intervals, usually twice, or four times a year, the profits are divided among the stockholders, except what may be reserved for the needs of the business. The sum paid is called a *dividend*, and is usually expressed as a rate per cent of the face value of the share.

NUMBER

SHARES

THE AMERICAN DISTRICT TELEGRAPH COMPANY

of Michigan, Limited.

This Certifies that Alfred Morgan is the owner of One hundred Shares of One Hundred Dollars each fully paid of the Capital Stock of THE AMERICAN DISTRICT TELEGRAPH COMPANY OF MICHIGAN, LIMITED.

Transferable only on the books of the Company and in accordance with the rules and Regulations of said Company, on surrender of this Certificate.

[SEAL]

IN WITNESS WHEREOF, The Chairman and Secretary of said Company by authority of the Board of Managers, have hereunto subscribed their names and caused the Seal of said Company to be hereto affixed, this First day of June A.D. 1905.

Henry Schantz SECRETARY.

James Perkins CHAIRMAN.

Lithographed by Donaldson Lithographing Company

Sale of Stock. When a stockholder wishes to withdraw his capital from the business, he sells his stock to any purchaser he may find. A share of stock may be sold for its face value, or for more or less, according to the state of success of the business and other considerations. If it sells for its face value, it is said to be **at par,** otherwise, **above** or **below par.**

Just as there are "commercial exchanges" in the large cities to facilitate the purchase or sale of crops, so there are "stock exchanges" to facilitate the purchase and exchange of stocks. The stock broker charges a commission of $\frac{1}{8}$ of 1% on the *par value* for either buying or selling.

When the price of stock is quoted 90, 85, 45, 37½, or 284¼, it means that a share whose par value is $100 can be bought for $90, $85, and so on.

1. What must one *add* to each quotation to find the cost of a share, including brokerage? What must one *subtract* to find what the sale of a share would net the seller?

Written.

What is the cost of the following purchases of stock:

2. 17 shares of Baltimore and Ohio Railway stock at 105?
3. 150 shares of Illinois Central Railway stock at 128?
4. 2,000 shares of St. Louis and S. W. stock at 128?
5. 5,000 shares of Texas Pacific Railway stock at $41\frac{1}{2}$?
6. What did the owner of the stock in each of the above Exercises realize from its sale, after having paid the broker?

How much was realized from the following sales:

7. 25 shares of United States Steel at 98?
8. 125 shares of Amalgamated Copper at 75?
9. 75 shares of Pennsylvania Railway at $108\frac{1}{2}$?
10. 100 shares of Grand Trunk Railway at $112\frac{1}{2}$?
11. The par value of a share of stock in each of the quotations above is $100; which stocks were bought or sold above par? Below par?

Oral.

1. Mr. Richmond owned 12 shares of New York Railway stock; he received an annual dividend of \$6 a share; what was his yearly income from these shares?

2. Mr. Marsh received a semiannual dividend of 4% on 10 shares of mining stock; what was his annual income?

3. Mr. A owns 10 shares of stock on which a dividend of 3% has been declared; how much does he receive?

4. What is the annual income from 160 shares of stock which pays a quarterly dividend of $1\frac{3}{4}\%$?

Written.

5. Mr. Field bought 100 shares of Chicago and Great Western Railway stock at $89\frac{7}{8}$; how much did he pay for the stock including brokerage? It paid an annual dividend of 4%; how much did he receive in dividends?

6. How many dollars did Mr. Field receive for every \$90 invested? How much was this on each dollar?

Income on Investment. The amount of income received per dollar of investment is called the *rate of income on the investment.*

7. If stock paying an annual dividend of 6% costs 120 (brokerage included), how many dollars are received each year from an investment of \$120?

8. What is the rate of income in Exercises 5 and 7?

9. If Diamond Match stock sells for 140 and pays an annual dividend of 10%, find the rate of income?

10. What is the rate of income on an investment in 10 shares of stock paying 4% annually and bought at 80?

11. If an annual dividend of 6% is paid on 200,000 shares of stock, how much is paid in dividends each year?

12. A company, the par value of whose stock is \$50, declares an annual dividend at the rate of \$4.75 a share; what is the rate per cent of the dividend?

Preferred and Common Stock. Sometimes the capital stock of a corporation is divided into two kinds: *preferred* and *common*. A dividend is usually guaranteed on the preferred stock, and this dividend must be paid before any dividend whatever may be paid on the common stock.

Oral.

1. A carpet company organized with a capital of $1,000,000; how many 100-dollar shares of stock were there?

2. The members of the firm subscribed for $\frac{1}{2}$ of the stock; how many shares were sold to other investors?

3. 2,000 shares were preferred stock; how many shares of common stock were there?

4. What sum was required annually to pay the dividends on the preferred stock at 5%?

5. At the end of the first year a 2% dividend was declared on the common stock; how much was this a share? What sum was paid as dividends on the common stock?

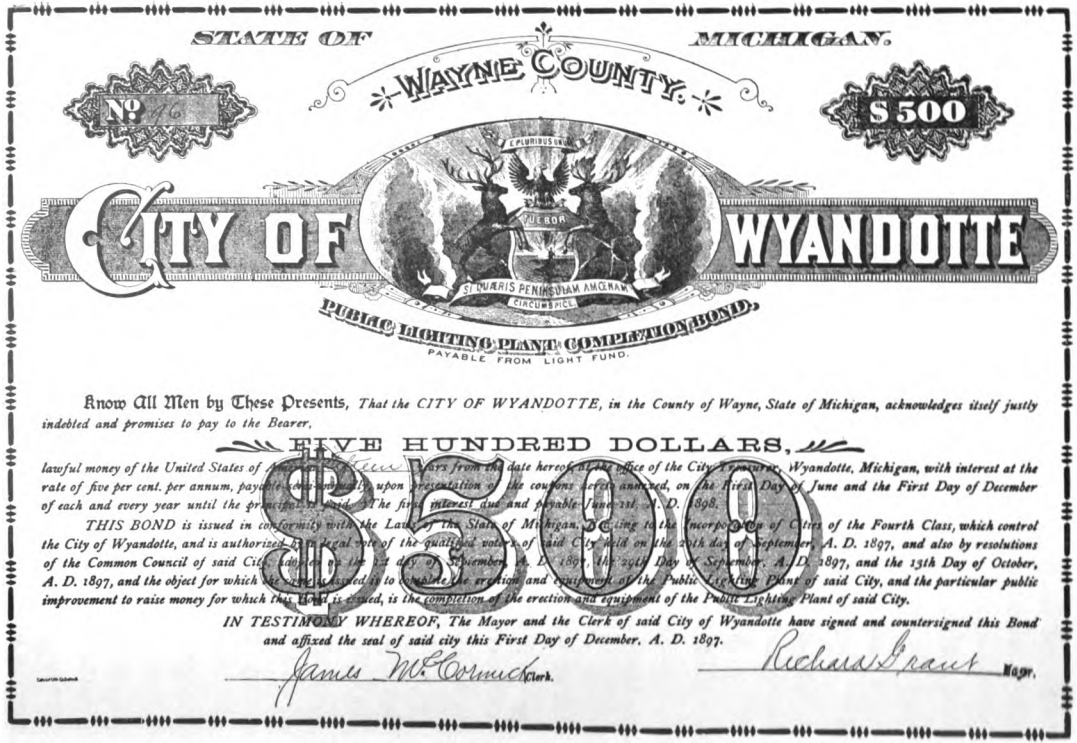
6. Find the cost at the opening price of 100 shares of each of the first 5 stocks quoted in this list.

7. Find in the table cases where both the preferred and the common stock of the same company is quoted (pr. means preferred). Which is worth the more? Find the cost of 100 shares of each at the highest price of the day.

8. Make and solve 20 problems from these quotations.

Sales and range of prices:

	Sales.	Open.	High.	Low.	Close.
Amalgamated...	242600	57	57⅜	56⅛	56⅝
Atchison	10400	80¾	81	80⅛	80⅝
Atchison pr.....	300	98½	98⅝	98½	98½
B. & O.....	17400	86¾	87⅛	86⅛	86⅝
Can. Pacific.....	1300	127	127⅞	127	127⅛
C. of N. J.....	100	168¾	168¾	168¼	168¾
C. & O.....	14500	38⅜	39⅛	38⅜	38⅞
C. & A.....	300	41⅞	41¾	41	41¼
C. & A. pr.....	100	83	83	83	83
C., M. & St. P....	13900	155	155¾	153⅞	154
C. & N. W.....	200	186⅞	187	186⅞	187
Chi. Term.	400	6⅛	6⅜	5¾	5¾
Chi. Term. pr....	2600	14	14	11⅛	12⅜
Chi. Un. Trac....	4400	7¼	8¼	7¼	7⅞
Chi. Un. Trac. pr.	600	39⅝	40	39⅛	40
Erie	73920	27⅜	28⅛	27⅜	28¼
Erie 1st pr.....	8335	64⅜	65⅛	64¾	64¾
Erie 2d pr.....	3000	40⅛	41⅛	40⅛	40⅝
Hock. Valley....	700	83	83⅜	83	83⅜
Hock. Valley pr.	1200	90⅛	91	89⅛	89⅞
Illinois Central..	1045	137¾	137¾	137¼	137¼
Iowa Central....	100	21⅞	21⅞	21⅞	21⅞
Iowa Cent. pr....	200	42	42¼	42	42¼



STATE OF MICHIGAN.

WAYNE COUNTY.

No. 16

$500

E PLURIBUS UNUM

VI ET VIRTUTE

SI QUAERIS PENINSULAM AMOENAM CIRCUMSPICE

CITY OF WYANDOTTE

PUBLIC LIGHTING PLANT COMPLETION BOND.

PAYABLE FROM LIGHT FUND.

Know All Men by These Presents, *That the CITY OF WYANDOTTE, in the County of Wayne, State of Michigan, acknowledges itself justly indebted and promises to pay to the Bearer,*

FIVE HUNDRED DOLLARS,

lawful money of the United States of America, [illegible] years from the date hereof, at the office of the City Treasurer, Wyandotte, Michigan, with interest at the rate of five per cent. per annum, payable semi-annually upon presentation of the coupons hereto annexed, on the First Day of June and the First Day of December of each and every year until the principal is paid. The first interest due and payable June 1st, A. D. 1898.

THIS BOND is issued in conformity with the Laws of the State of Michigan, relating to the Incorporation of Cities of the Fourth Class, which control the City of Wyandotte, and is authorized by a legal vote of the qualified voters of said City held on the 20th day of September, A. D. 1897, and also by resolutions of the Common Council of said City adopted on the 7th day of September, A. D. 1897, the 20th Day of September, A. D. 1897, and the 13th Day of October, A. D. 1897, and the object for which the debt is created is to complete the erection and equipment of the Public Lighting Plant of said City, and the particular public improvement to raise money for which this Bond is issued, is the completion of the erection and equipment of the Public Lighting Plant of said City.

IN TESTIMONY WHEREOF, The Mayor and the Clerk of said City of Wyandotte have signed and countersigned this Bond and affixed the seal of said city this First Day of December, A. D. 1897.

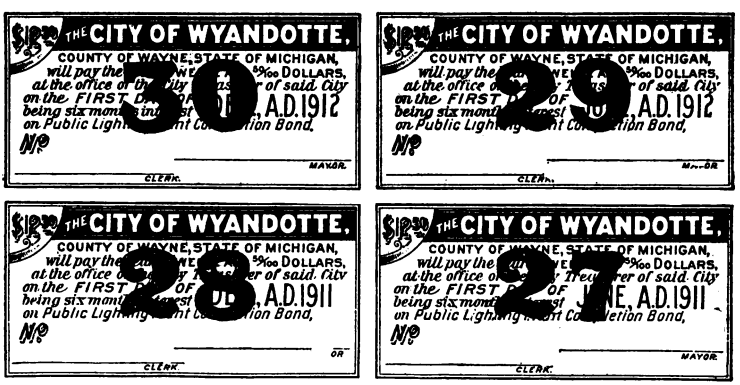
James McCormick Clerk. Richard Grant Mayr.

Bonds. When the government (national, state, county, city) or a corporation borrows large sums of money for a long time, the acknowledgment of the receipt and the promise to repay is usually called a *bond*. A bond is shown on page 174. A bond is usually issued for each $1,000 borrowed (sometimes less).

Par Value. The amount of the bond is called its *par value*.

Coupons. Bonds bear interest at a specified rate. This is usually paid upon presentation of certificates called *coupons*, attached to the bonds.

The coupons are promissory notes, which, if not paid when due, bear interest at the legal rate.



1. What is the face of the bond shown on page 174? What is the annual interest at 5%?

2. The above coupons are the last 4 of the 30 coupons of this bond. Each represents the interest for 6 months. What was the whole amount of interest on the bond?

3. If the last five coupons were not paid until the last one came due, how much was then due, the coupons bearing interest at 5%?

Difference between Bonds and Stock. Bonds differ from stock in that the bonds are secured by the property of the company and bear a permanent rate of interest.

The interest must be paid on the bonds before any dividend can be paid on the stock. A bondholder is a creditor; a stockholder is a part owner. Bonds, like stocks, are bought and sold in the market. The bond and interest coupons are payable to the bearer, unless a record of ownership is kept by the corporation issuing the bonds. Such bonds are called **registered** bonds. The practise of registering bonds lessens the danger of theft and fraud.

Brokerage. The broker's commission for buying or selling bonds is $\frac{1}{8}$ of 1% of the *par value*.

Amounts bought and sold are stated in par value, and the price is quoted at a rate per $100 of par value. The rate of interest is usually added.

Thus, U. S. 4's 142 means that United States bonds bearing 4% interest on par value are sold at $142 for each $100 of par value.

1. What is the cost of $3,000 (par value) Erie canal bonds at 101?

PLAN. 1. Each $100 of par value costs $101.
2. Therefore, $3,000 of par value cost 30 × $101, or $—.
3. Brokerage, $\frac{1}{8}$ of 1% of par value, $—.
4. Total cost = $—.

2. What is the cost of $15,000 bonds of the city of Chicago at 105?

3. Find the proceeds from each of the following sales:

$5,000 Central Union Gas Co. of N. Y. 1st mtge. 5% bonds. 108½	$20,000 Northern Pacific Gt. Northern R. R. (C., B. & Q. coll.) 4% bonds..... 94⅛
$10,000 Union Gas Light Co. of Kings Co. 1st mtge. 6% bonds..... 106⅜	$6,000 Southern R. R. 1st consld. mtge. 5% bonds..... 115⅜

4. Find the annual income from each purchase named in Exercise 3.

5. Find the brokerage on each of the above sales.

6. If convenient, bring newspaper clippings giving prices of bonds and solve other problems.

Oral.

1. What is meant by the stock of a corporation? What is the usual par value of stock? Name some stocks quoted above par in the table on page 173.

2. What is a dividend? On what value of the stock is it declared?

3. Where are stocks usually bought and sold and through whom? What is the fee for buying or selling called?

4. What is compound interest? What banks pay the equivalent of compound interest?

5. How is compound interest usually computed?

6. What is a bond? What is usually attached to a bond to represent the interest due?

7. What is meant by registered bonds? Why are bonds registered?

8. Where are bonds usually bought and sold? What must one add to the quoted price of a bond to find its cost?

9. What is the cost of 500 Metropolitan bonds (par \$100) quoted at $109\frac{7}{8}$?

10. A man bought 10 bonds (par \$100) quoted at $149\frac{7}{8}$ and paying an annual dividend of $4\frac{1}{2}\%$; what was his rate of income on the investment?

Written.

11. Mr. Johnson bought 100 shares of Pennsylvania Railroad stock at 130. How much did he pay his broker? How much did the seller receive from his broker?

12. If New York Central stock pays a dividend of 5%, what is the income from 238 shares?

13. What is the income from 250 shares of stock paying a dividend of $4\frac{1}{2}\%$?

14. A man purchased some stock at \$110 a share which paid a dividend of $5\frac{1}{2}\%$; what was the rate of income on the investment?

XIX

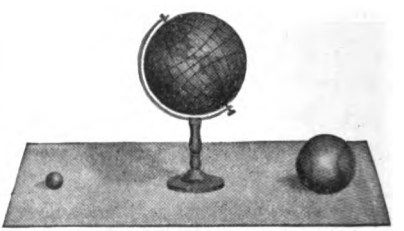
FORM STUDY AND MEASUREMENT

SPHERES

Sphere. If a circle be turned about a diameter as an axis, the surface described is called a **sphere.** The center of the circle is the **center** of the sphere. The space inclosed is called the **volume** of the sphere.

The straight line connecting any point of the sphere with the center is called a **radius** of the sphere.

Any two radii lying in one straight line form a **diameter** of the sphere.



Marbles, balls, and globes are examples of spherical forms.

Oral.

1. Name other examples of spheres.

2. How do the distances from the center to all points on the sphere compare?

Circles. When a flat surface or plane cuts a sphere, the line of cutting is a circle. When the cutting plane passes through the center of the sphere, the circle is called a **great circle**; otherwise, it is called a **small circle.**

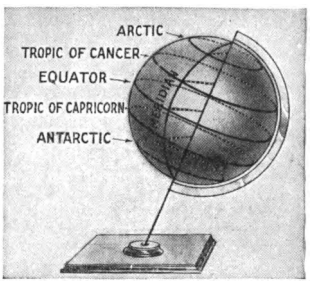
Such circles may be seen by immersing some round object, as an orange or a croquet ball, in water to various depths, also by actually cutting spherical bodies, for example, balls of clay.

The two portions into which a sphere is divided by any cutting plane are called **segments.** When the cutting plane passes through the center, the segments are equal and are called **hemispheres.**

3. What is the shape of the dome of the capitol?

1. Taking the equatorial radius of the earth to be 4,000 mi., what is the length of the equator?

2. How many degrees are there in any circle? What is the length (to the nearest mile) of 1° (one degree) on the equator?



3. The radii of the tropics may be taken to be 3,600 mi., and of the polar circles 1,400 mi.; how many miles are there in 1° on each circle?

Meridians. The circles whose diameters lie in the axis connecting the north and south poles of the earth are called *meridianal circles.* Each half circle from pole to pole is called a *meridian.*

4. Are meridians large or small circles? Find the length of a meridianal circle.

5. What is the length of a meridian? Of 1° on a meridian?

In the study of geography it is seen that the location of a point on the earth's surface may be indicated by stating its distance north or south of the equator and east or west of a given meridian.

Latitude. Distance in degrees north or south of the equator is called *latitude.*

To say that a place is in latitude 42° N. means that it is 42° north of the equator.

Longitude. Distance measured in degrees east or west of a given meridian is called *longitude.*

The meridian of Greenwich, a suburb of London, is usually taken. That of Washington is sometimes used.

To say that a place is in longitude 87° W. means that it is 87° west of the meridian of Greenwich.

6. If you walk straight north, how does your latitude change? Your longitude? If you walk east? West?

Angular Measure. 1° is divided into 60 equal parts called **minutes,** and 1 minute is divided into 60 equal parts called **seconds.**

Notation. Minutes and seconds are denoted by ′ and ″ respectively.

Thus, 12 degrees, 14 minutes and 17 seconds would be written 12° 14′ 17″.

1. Write similarly 40 degrees, 30 minutes and 59 seconds; 84 degrees, 55 minutes and 43 seconds.

2. How many seconds are there in 1′? In 2′? In $\frac{3}{4}$′?

3. How many minutes are there in $\frac{1}{2}$°? In $1\frac{1}{2}$°?

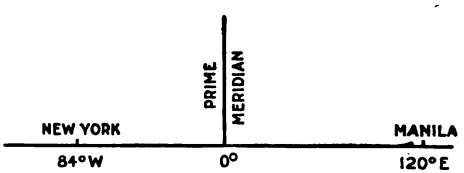
4. Add 30° 16′ 23″ and 37° 50′ 19″.

30°	16′	23″
37	50	19
68°	6′	42″

Numbers expressed in degrees, minutes, and seconds are best added by columns, and the usual tests applied to the work of each column. Similarly for subtraction.

5. The longitude of New York is 73° 58′ 25.5″ west and that of Chicago is 87° 36′ 42″ west; what is the difference in longitude between these places?

6. Find from the figure the difference in longitude between Manila and New York City.



When one place is in east longitude and the other in west, the difference in longitude between the two places is found by adding.

7. Find the difference in longitude between Berlin and each of the other places in the first column of the table:

PLACE.	LONGITUDE.	PLACE.	LONGITUDE.
Berlin, Germany..	13° 23′ 43.5″ E.	Manila, Philippines..	120° 58′ 00″ E.
Dublin, Ireland...	6 20 30 W.	Peking, China	116 27 00 E.
Honolulu, Hawaii.	157 51 48 W.	New York	73 58 25.5 W.
Paris, France	2 20 15 E.	Chicago	87 36 42 W.
Tokyo, Japan.....	139 42 30 E.	San Francisco.....	122 25 40.8 W.

8. Make and solve 10 other problems from the table.

Oral.

1. How many hours are required by the earth to make one rotation on its axis? Through how many degrees does a point on the equator pass in 24 hours? In 1 hour?

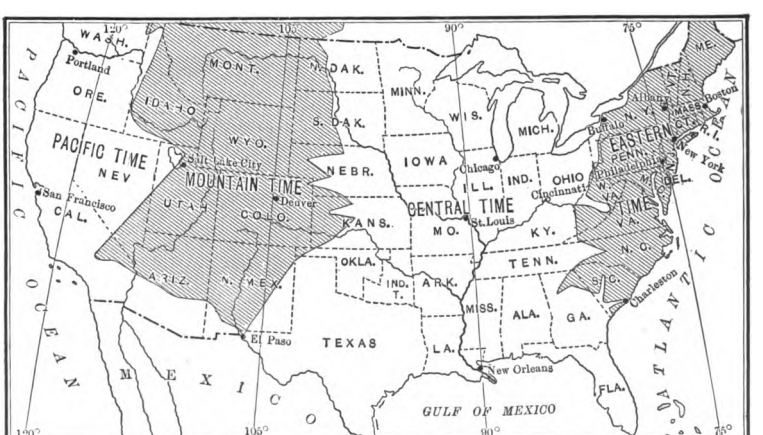
2. In which direction does the earth rotate?

3. When it is sunrise in New York City, is it before or after sunrise in Boston? In Buffalo? In San Francisco?

4. How many hours elapse while a point of the earth's surface turns through 360°? 180°? 90°? 30°? 15°?

5. When it is noon in Chicago, what time is it in a place 15° west of Chicago? In one 15° east of Chicago?

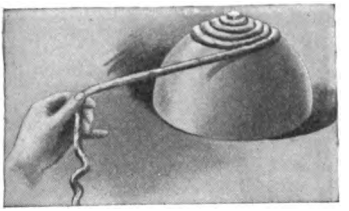
All places east of a given point have later time, all west of it have earlier time. This caused much confusion in railway travel, hence the country has been divided into sections corresponding to standard meridians, the time of the meridians being taken as the time throughout the sections. This is called standard time. The meridians used are 75°, 90°, 105°, 120°. The map shows the meridians and the corresponding time belts.

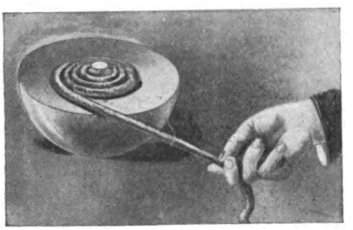


6. When it is noon, standard time at New York, what is the time at Boston? Denver? Philadelphia? Chicago?

7. Make and solve 10 other problems.

By winding a hemisphere closely with a string and then winding this string on the base of the hemisphere, it is found that twice as much string is required to cover the convex surface as to cover the base.





Oral

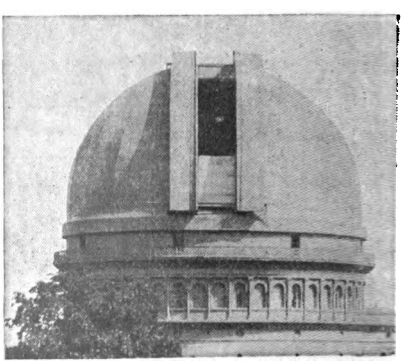
1. What is the area of the base? How does the area of the hemispherical surface compare with this area? How does the area of the sphere compare with it?

2. How does the radius of a sphere compare with that of one of its great circles? What represents the area of a circle of radius r?

3. What does $4\pi r^2$ express when r stands for the radius of a sphere?

4. Using the letter π, state the area of a sphere of radius 3 in.; 6 in.; 4 ft.; 1 yd.

Thus, for radius 3 in. the area is $36\ \pi$.



Written.

5. Using 3.14 as the value of π, compute the area of each sphere of Exercise 4.

6. The radius of the base of the dome of this observatory is 45 ft. Find the perimeter of the base.

7. What is the cost of painting the outer surface of the dome at 2ϕ a square foot, regarding it as a hemisphere?

Oral.

1. Name three bodies that are spherical in form.

2. What is a great circle of a sphere? A small circle?

3. State which of the following are great and which small circles: equator, meridians, tropical and polar circles.

4. Explain the meaning of east and west longitude.

5. What is the difference in longitude between two places:

One	60°	west	and the other		95°	west.
"	75°	"	"	"	122°	"
"	0°	"	"	"	150°	east.
"	15°	east	"	"	90°	west.
"	150°	"	"	"	150°	"
"	180°	"	"	"	180°	"

6. Find the difference in longitude between Paris, 2° 20′ 15″ E. and New York 73° 58′ 25″ W.

7. When it is noon in Chicago is it before or after noon in New York? In San Francisco? In Salt Lake City? In Charleston? In Boston?

8. When it is noon on the 90th meridian west, what time is it at 75° W.? At 120° W.? At 105° W.? At 15° E.? At 0°?

9. What are the meridians used for standard time in the United States? How many hours difference is there in the standard time of Boston and San Francisco?

10. How is the area of a sphere found from its radius?

11. What is the area of a sphere whose radius is 1 ft.?

Written.

12. A hemispherical skylight is 12 ft. in diameter; how many square feet of glass are there in its surface, not allowing for sash?

13. A hemispherical ceiling 25 ft. in diameter is covered with steel at 30¢ a square foot; what did it cost?

XX

USE OF LETTERS—ALGEBRA

ADDITION AND SUBTRACTION

Oral.

1. $3a + 4a = ?$ If a is 1, what is the value of $3a$? Of $4a$? Of $3a + 4a$?

2. $6b + 5b = ?$ If b is 1, what is the value of $6b$? Of $5b$? Of $6b + 5b$?

3. If $a = 1$ and $b = 1$, what is the value of $3a + 6b$?

Find the value of each of these expressions, when each letter $= 1$:

4. $a + 2b$. 5. $2a + 2b$. 6. $3a + b$.

7. $a + b + c$. 8. $a + 2b + c$. 9. $2a + 3b + 3c$.

10. $b + c + 2a$. 11. $b + d + 3c$. 12. $c + 4d + 2a$.

13. Find the numbers to fill the blanks in the sum; what is the sum of $3a + 4b$ and $2a + 6b$?

14. When $a = 1$ and $b = 1$, what is the value of each addend in Exercise 13? What is the sum of these values? What is the value of the sum? Explain how to test the work of adding.

$3a + 4b$	7
$2a + 6b$	8
$(\ )a + (\ )b$	15

Add and test by letting $a = b = c = 1$:

15. $a + b + c$ / $2a + b + 3c$

16. $a + 2b + c$ / $a \quad\quad + 6c$

17. $6a + b + c$ / $2b + c$

18. Find the numbers to fill the blanks in this subtraction. Let $a = 1$, $b = 1$; how is the work tested?

$4a + 7b$	11
$2a + 5b$	7
$(\ )a + (\ )b$	4

Written.

Subtract and test:

19. $6a + 10b$ / $2a + 5b$

20. $2a + 5b + 4c$ / $a + b + c$

21. $3x + 7y + 11z$ / $2x + y + 4z$

Add and test:

1. $a + 3b$ / $10a + 11b$
2. $5a + 9b$ / $5a + 3b$
3. $16a + 4b$ / $4b + 16a$
4. $a + b + c$ / $m + 6c$
5. $4a + c$ / $2a + b + c$
6. $6a + b$ / $3a + 2b + c$
7. $m + n + p$ / $2m + 5n + 9p$
8. $45m + 3n + p$ / $5m + 2n + p$
9. $6m + 4n + p$ / $4m + 6n + 9p$

Subtract and test:

10. $5a + 6b$ / $a + 3b$
11. $10a + 5b$ / $9a + 5b$
12. $17a + 9b$ / $17a + b$
13. $2a + b + 7c$ / $a + b + 5c$
14. $10a + 6b + 5c$ / $5a + b + 5c$
15. $16a + 2b + 2c$ / $12a + b$
16. $25 + n + 7p$ / $5 + n + p$
17. $45m + 3n + 12p$ / $5m + n + 2p$
18. $17m + 6n + 2p$ / $9m + n + 2p$

19. A fruit grower picked $2a$ bu. of apples, $3b$ bu. of peaches, and $4c$ bu. of plums in one season; how many bushels of fruit did he pick?

20. The same man picked $4a$ bu. of apples, b bu. of peaches, and $3c$ bu. of plums the next season; how many bushels of fruit did he pick? How many bushels did he pick in the two seasons?

21. For a house a builder used of three kinds of lumber $50a$ ft., $10b$ ft., $12c$ ft. respectively; for another house he used of the same kinds $75a$ ft., $15b$ ft., and $10c$ ft.; how many feet did he use in all?

22. A grain dealer had $8m$ bu. of oats, $10n$ bu. of wheat, and $7p$ bu. of rye, and sold $6m$ bu. of oats, $5n$ bu. of wheat, and $7p$ bu. of rye; how many bushels of grain had he left?

Exponents. The number written above and at the right of a number to show how many times it is taken as a factor has already been called an *exponent* (page 17).

Thus, in c^3, the 2 shows that c^3 means cc, also in 3^2y^3 the 2 and 3 show that $3^2y^3 = 3 \times 3 \times y \times y \times y$.

Write the equivalent forms by using exponents:

1. 9 (or 3×3).
2. 25.
3. 49.
4. 100.
5. 8 (or $2 \times 2 \times 2$).
6. 16.
7. 27 (or $3 \times 3 \times 3$).
8. 81.
9. bb.
10. ccc.
11. $aayyy$.
12. $8aa$.
13. $16\,bbb$.
14. $sssss$.
15. $100aabb$.

Indicate by use of exponents the number of:

16. Days in 7 weeks.
17. Seconds in an hour.
18. Things in a gross.
19. Cu. in. in a cu. ft.
20. Months in 12 years.
21. Years in 100 centuries.

In the following problems, express the result as a power, then perform the multiplication in the first one only of each set:

22. A man walked 7 miles daily for 7 weeks; how many miles did he walk? How many miles did he walk in $7w$ weeks?

23. How many things are there in 12 gross? In g gross? In $12g$ gross?

24. A man buys a dozen magazines at $12¢$ each, monthly for 12 years; what is his total expenditure for this purpose? What is his expenditure in y years? In $24y$ years?

25. A packing box contains 10 cases of goods; each case contains 10 rolls of cloth; each roll contains 10 yd., and each yard is worth $10¢$; what is the value of the contents of the box? What is the value of c boxes? Of $100c$ boxes?

Multiply:

26. $4a \times 3a$
27. $5b \times 2b$
28. $8c^2 \times 3c$
29. $4x^3 \times 2x^2$

Monomials. An expression which does not contain the sign of addition (+) or of subtraction (−) is called a *monomial.*

Thus, 7, $\frac{26}{5}$, $\sqrt{18}$, a, a^2c, $\frac{ab}{4^3}$, $xy \div a$ are monomials.

Binomials. The indicated sum or difference of two monomials is called a *binomial.*

Thus, $7 - 4$, $8 + 6$, $a + b$, and $c - 5d^2$ are binomials.

Trinomials. The indicated combination of three monomials by addition or subtraction is called a *trinomial.*

Terms. The numbers separated by the signs + or − are called *terms* of the expression.

Thus, $6 + 8 + 4$, $6 + 8 - 4$, $12 - 7 - 3$, $a + 5b - c$ are trinomials. 6, 8 and 4 are terms of $6 + 8 + 4$, and a, $5b$, c of $a + 5b - c$.

Parentheses. The parenthesis, ( ), may be used to group several terms into one expression.

The following examples illustrate the method of reading expressions containing parentheses:

$4\,(8 - 3)$ is read "4 times the binomial, 8 minus 3."
$a\,(b - c + d)$ " " "a times the trinomial, b minus c plus d."
$(a + b)\,(4 - 3)$ " " "The binomial, $a + b$, times the binomial, $4 - 3$."
$(8 + 6 - 2) \div 4$ " " "The trinomial, $8 + 6 - 2$, divided by 4."
$(b + 6)^2$ " " "The square of the binomial, $b + 6$.

Oral.

Read the following:

1. $(17 + 15) \div (9 - 3 + 10)$.
2. $(12 - 7)^2$.
3. $(a + b) - (c - d)$.
4. $(x + y) - c$.
5. $(a + b - c) + (b - d)$.
9. $(x + 1)^2$.
7. $(a + b)\,(b - c)^2$.
8. $(a^2 + b^2)^2$.

Select the monomials; the binomials; the trinomials:

9. $a + b - c$.
10. $4x^2 + 7$.
11. $a + b + c + d$.
12. $x - y + z + w$.
13. m.
14. $a + 5b - c$.
15. $2g - 5 + x^2$.
16. $gt^2 + a$.
17. $5x^2 + ay^3$.

Multiplication by Literal Expressions is performed like the multiplication of ordinary numbers.

To multiply by 32 we multiply by 30 and by 2 and add the results. To multiply by $a + b$ we multiply by a and by b and add the results. The following examples illustrate the process:

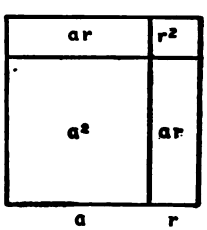
$$\begin{array}{r} a + 7b \\ a + b \\ \hline a^2 + 7ab \\ \quad ab + 7b^2 \\ \hline a^2 + 8ab + 7b^2 \end{array} \qquad \begin{array}{r} 8 \\ 2 \\ \hline \\ \\ \\ 16 \end{array} \qquad \begin{array}{r} a + 2 \\ a + b \\ \hline a^2 + 2a \\ \quad + ab + 2b \\ \hline a^2 + 2a + ab + 2b \end{array} \qquad \begin{array}{r} 3 \\ 2 \\ \hline \\ \\ \\ 6 \end{array}$$

The numbers at the right are the values of the corresponding expressions when $a = 1$, $b = 1$. These serve to check the work. Thus, in the first $2 \times 8 = 16$, and in the second $2 \times 3 = 6$.

Multiply and test the work by giving values to the letters:

1. $3a + x$ by $a + b$
2. $4x + 5$ by $a + x$
3. $2 + a$ by $b + x$
4. $y + 3$ by $3y + 2$
5. $x + 1$ by $x + 1$
6. $n + 2$ by $n + 2$
7. $z + 5$ by $z + 5$
8. $x + y$ by $x + y$

9. Show that
$$\begin{array}{r} a + r \\ a + r \\ \hline a^2 + 2ar + r^2 \end{array}$$



10. Show from the figure that the square on a + r is the square on a plus the square on r plus twice the rectangle of a and r.

11. Replace a by 30 and r by 2 throughout the work of Exercise 9. What number have you squared?

12. What is the result if $a = 40$, $r = 3$? What is the square of 43?

The result of Exercise 9 says: The square of the sum of any two numbers is the square of the first plus twice the product of the first and second, plus the square of the second.

13. Square 17 by writing it $10 + 7$.

14. Find similarly: 24^2; 13^2; 41^2; 25^2; 42^2; 33^2.

Oral.

State the quotients in the following cases of division:

	1.	2.	3.	4.	5.
Dividend:	$36x$	15	$15a$	$14b$	xy
Divisor:	9	5	5	7	y

6. How may the dividend be found from the divisor and quotient?

Find the numbers to fill the blanks:

	7.	8.	9.	10.	11.
Dividend:	—	—	$21ay$	$42t$	$18ab$
Divisor:	$3a$	$9y$	—	$7t$	$9a$
Quotient:	$4b$	$2x$	$7a$	—	—

12. Divide $a^2 + 3ab + 2b^2$ by $a + b$.

Divide the first term of the dividend by the first term of the divisor; the quotient is a. Multiply the entire divisor by a and subtract, as in the case of numbers. Divide the first term of the remainder by the first term of the divisor again; the quotient is $2b$. Multiply and subtract, as in the case of numbers.

$$\begin{array}{r} a + 2b \\ a+b \,\overline{)\, a^2 + 3ab + 2b^2} \\ a^2 + ab \\ \hline 2ab + 2b^2 \\ 2ab + 2b^2 \\ \hline \end{array}$$

Multiply the quotient by the divisor to test the work.

Written.

Divide and test:

13. $c^2 + 2cd + d^2$ by $c + d$.

14. $a^2 + 4ab + 4b^2$ by $a + 2b$.

15. $mn + np + nq$ by n.

16. $3a^2 + 7ab + 2b^2$ by $3a + b$.

17. $m^2 + 3mn + 2n^2$ by $m + n$.

18. $x^2 + 5xy + 6y^2$ by $x + 2y$.

19. $4a^2 + 4ab + b^2$ by $2a + b$.

20. $mp + p^2 + pq$ by p.

Multiply:

21. $x + 10$ by $x + 20$.

22. $a + 1$ by $b + 1$.

23. $2a + 1$ by $2b + 3$.

24. $3x + y$ by $3y + x$.

Factoring. Thus far the factors have been given and the product sought. But the product may be given and it may be required to find the factors, that is, a set of numbers whose product is the given number. This is called *factoring* the given expression. It may usually be done in various ways.

Thus, $28 = 4 \times 7$, or 2×14, or $2 \times 2 \times 7$, or 28×1.
Also, $4ax = 4 \times a \times x$, or $4a \times x$, or $a \times 4x$, or $2 \times 2a \times x$, or $4 \times ax$.

Integral Factors. An *integral factor* is one which is not in the form of a fraction. All the factors above are integral factors.

An integral factor involving no letters is a whole number. If it involves letters, they may represent fractional values. But the *form* of the factor must not be that of a fraction.

Thus, $4a$ is an integral factor, but $\frac{4a}{3}$ is not. In what follows we shall consider only integral factors, and shall use simply the word *factors*.

Oral.

Factor each of the following expressions in two ways:

1. $6a$.	2. $15axy$.	3. $7ay$.
4. $6abc$.	5. $8a^2b$.	6. $14br^2$.
7. $14a^2$.	8. $48x$.	9. $21a^3x^2$.

Regarding 3a *as one of two factors, what is the other factor of each of the following expressions:*

10. $6a$.	11. $9ab$.	12. $30axy$.
13. $3ac$.	14. $12az$.	15. $21a^2$.
16. $15a^3$.	17. $24ay^2$.	18. $15ax^4$.

Regarding x *as one of two factors, what is the other factor of each of the following expressions:*

19. $4x$.	20. x^2.	21. $1x$.
22. $10ax$.	23. $10axy$.	24. $6a^2x$.
25. $2bx$.	26. $5x^2z$.	27. $7ax^2$.
28. mx.	29. x^3.	30. $2rx^4$.

The equal factors of numbers may be indicated by exponents.

Thus, $125 = 5 \times 5 \times 5$, or 5^3.

Also, $126 = 2 \times 3 \times 3 \times 7$, or $2 \times 3^2 \times 7$.

1. Express the factorable numbers from 1 to 150 as products of prime factors.

2. What is one of the two equal factors of a number called?

Square Root by Factoring. The square root of perfect squares may be found by factoring.

Thus, $196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2$. Hence the square root of 196 is 2×7, or 14.

The symbol $\sqrt{\ }$ means "the square root of." Thus, $\sqrt{196} = 14$.

Similarly, $400 = 2^4 \times 5^2$, hence $\sqrt{400}$ is $2^2 \times 5$, or 20.

Also, $169a^2b^4 = 13^2a^2b^4$, hence $\sqrt{169a^2b^4}$ is $13ab^2$.

Oral.

State the square root of:

3. 25. 4. a^2. 5. b^6.

6. 144. 7. 81. 8. a^2b^2.

9. $16a^4$. 10. $49x^2$. 11. $64b^2$.

12. 256. 13. $225c^4$. 14. y^6.

15. $121m^4$. 16. $100p^8$. 17. $16a^4c^6$.

When there are so many factors that it is not easy to separate them into two equal groups by inspection, the factors may first be written out in full, and then grouped.

Thus, $7056 = 4 \times 4 \times 3 \times 3 \times 7 \times 7$

$= (4 \times 3 \times 7)\,(4 \times 3 \times 7) = 84^2$.

Therefore, $\sqrt{7056} = 84$.

The work of finding the factors may be arranged thus:

4) 7056
4) 1764
3) 441
3) 147
7) 49
7

Written.

18. Find $\sqrt{625}$; $\sqrt{11{,}025}$; $\sqrt{1024b^8}$.

19. Find $\sqrt{256a^4b^4}$; $\sqrt{3025a^2}$; $\sqrt{169a^4b^6}$.

Order of Operations. When not otherwise indicated multiplications and divisions are performed before additions and subtractions.

Thus, $3 + 7 \times 2$ is taken to mean $3 + 14$, or 17.
Similarly, $4 + 12 \div 2$ means $4 + 6$, or 10,
and $5 + 14 \div 7 - 3 = 5 + 2 - 3 = 7 - 3 = 4$.

Oral.

Perform the operations indicated:

1. $12 - 6 \div 3$.
2. $30 + 5 \times 8$.
3. $6 \times 4 - 3$.
4. $2 + 8 \times 4 - 3$.
5. $6 \times 5 - 12 + 7$.
6. $4 \times 9 - 10 \div 5$.
7. $8 + 2 \times 7$.
8. $12 - 16 \div 4$.
9. $40 - 36 \div 2$.
10. $52 \div 26 + 5$.
11. $14 \times 3 - 2 \times 10$.
12. $34 + 6 \div 2 - 17$.

Use of Parentheses. To indicate that additions and subtractions are to be performed first, the parenthesis is used.

Thus, in $(3 + 7) \times 2$, the parenthesis shows that 3 and 7 are first to be added, then the result multiplied by 2. It indicates that what is within the parenthesis is to be treated as a single expression.

Similarly, $(4 + 12) \div 2$ means $16 \div 2 = 8$.

Perform the operations indicated:

13. $(12 - 6) \div 3$.
14. $8 \times (30 + 5)$.
15. $(2 + 8) \times (4 - 3)$.
16. $6 \times (5 + 7 - 10)$.
17. $(16 - 12) \div 4$.
18. $(40 - 36) \div 2$.
19. $(84 + 6) \div (22 - 17)$.
20. $(8 + 12) \div (7 - 2)$.

Stating Problems. The parenthesis may often be used in stating problems.

Thus, the equation $2x = 9\,(44 - x)$ states the problem: "Of two numbers whose sum is 44, twice one is 9 times the other; find them."

Written.

Write the equation that states the following problem:

21. A rectangular lot is 60 ft. longer than it is wide, and 4 times the length is 10 times the breadth. Find the dimensions.

Oral.

1. Find: 21^2; 31^2; 14^2; 20^2; $5{,}000^2$; 51^2.

Regarding 2y *as one of two factors, state the other factor in each of the following expressions:*

2. $4ay$. 3. $20y$. 4. $2y^2$. 5. $30xy$. 6. $6ay^3$.

Perform the operations indicated:

7. $85+15\div5$. 8. $(85+15)\div5$. 9. $6\times8+3$.

10. $6\times(8+3)$. 11. $8a+12b\div4$. 12. $(8a+12b)\div4$.

Written.

Add:

13. $4a+2$ / $7a+9$ 14. $2x+6y$ / $3x+12y$ 15. $a+5b$ / $a+12b$ 16. $1+a$ / $2+5a$

Subtract:

17. $12m+16$ / $3m+1$ 18. $1+12q$ / $1+4q$ 19. $6x+27q$ / $5x+18q$ 20. $a+9b+5c$ / $a+2b$

Indicate by use of exponents:

21. The area of a 5-foot square.

22. The number of cubic feet in a cubic yard.

23. The price of 16 lb. of fine wire at 16¢ an ounce.

24. The number of inches in 36 yd.

Multiply:

25. $2x+1$ / $3x+4$ 26. $5a+b$ / $6a+3b$ 27. $a+m$ / $a+m$

Find the square root by factoring:

28. 3136. 29. 28224. 30. $9216a^2$. 31. a^4x^8.

32. Indicate by use of parentheses, that if twice the cost (c) of a lot be diminished by \$100 and the remainder multiplied by 3, the result will be the cost (h) of the house.

33. Find two numbers whose sum is 40 and whose difference is 6.

XXI

APPLICATIONS OF EQUATIONS

INDUSTRIAL PROBLEMS

Written.

1. In a factory there are three rates of pay: \$1.25, \$1.50, and \$2 a day. There are 10 men more receiving \$1.50 than \$2.00, and 18 more receiving \$1.25 than \$1.50. The daily pay-roll amounts to \$145; how many men are working at each rate?

PLAN. 1. Let x be the number of men working for \$2 a day.
2. Then $x + 10$ is the number working for \$1.50 a day.
3. And $x + 28$ is the number working for \$1.25 a day.
4. $2x + 1.5\,(x + 10) + 1.25\,(x + 28) = 145$. Why?
5. Therefore, $4.75x + 50 = 145$, and $x = —$.
6. 20 men work for \$2, — men for \$1.50, and — men for \$1.25.
7. Test.

2. A man inherited some money; he invested \$500 at 4% and the rest at 5%; his income was \$120 annually; how much did he inherit?

3. If an acre of land produces 20 tons of beets, and if beets yield 6% of their weight in sugar, how many acres of land are required to furnish the beets for a factory whose output of sugar is 190,000 lb. a year?

4. If a million pairs more of rubber shoes had been produced in the United States in 1900, there would have been one pair for every two persons of the 76 million inhabitants; find the number of pairs produced.

5. In a recent year the production of cane sugar in Cuba was twice that in the United States, 4 times that in Brazil, $\frac{8}{3}$ of that in the Sandwich Islands. As much was produced in Java as in the Sandwich Islands and the United States together. The total production in all of these countries was 2,400,000 tons. Find the production of each.

Written.

1. A train with average speed 35 mi. per hour starts from New York for Philadelphia, 90 mi. distant, at the same time that a train with average speed 30 mi. an hour starts from Philadelphia for New York; when will the trains pass each other?

PLAN. 1. Let x be the number of hr. after starting before they meet.
2. Then $35x$ is the distance traveled by the first train before they meet. 3. And $30x$ is what distance?
4. Therefore, $35x + 30x = 90$. Why? 5. Therefore, $x =$ —.

2. Two automobiles start from Chicago; one goes east at the rate of 20 mi. an hour and the other west at 15 mi. an hour; in how many hours are they 350 mi. apart?

3. A train 440 ft. long running at a speed of 20 mi. an hour requires $28\frac{1}{4}$ minutes to pass completely through the St. Gotthard tunnel in Switzerland; how long is the tunnel?

4. A train going from New York to Buffalo at the rate of 40 mi. an hour takes 2 hr. 12 min. longer than one going 50 mi. an hour; find the distance between these places?

5. The total number of miles of railway in the United States (to the nearest thousand) was 34,000 greater in 1900 than in 1890 and 5,000 greater than in 1899. The sum of all three numbers is 540,000. Find the mileage in 1900.

6. The number of freight locomotives in the United States in 1900 was 1,870 more than twice the number of passenger locomotives and 7,993 less than three times that number; find the number of each.

7. A baggage express company had 588 trunks to deliver. It delivered a certain number in the forenoon, $\frac{1}{12}$ of the whole in the afternoon and had 21 trunks undelivered at night; how many were delivered before noon?

8. A ship set sail from San Francisco for Manila, 8,800 mi. distant. In 4 days the distance yet to sail was 7 times that already sailed. Find the time required to make the trip.

Written.

1. The perimeter of a triangle is 38 in., the first side is 10 in. less than the second, and the third side is double the first. Find the length of each side.

PLAN. 1. Let x be the length of the first side in inches.
2. Then $x + 10$ is the length of the second side in inches.
3. And $2x$ equals what?
4. $x + x + 10 + 2x = 38$. Why? 5. Therefore, $x =$ —.
6. The first side is — in. What are the others?

2. The total width of the American and Horseshoe Falls at Niagara is 3,700 ft. The Horseshoe Falls are 10 ft. less than $2\frac{1}{2}$ times as wide as the American Falls. Find the width of each.

3. The two highest structures in the world are the Eiffel Tower at Paris and the Washington Monument. The Eiffel Tower is 429 ft. higher than the monument, but lacks 126 ft. of being twice as high. Find the height of each.

4. The three highest stone structures in the world are the Washington Monument, the City Hall of Philadelphia, and the cathedral at Cologne, Germany. The first is 18 ft. higher than the second and 31 ft. higher than the third. The sum of their heights is 1,616 ft. Find the height of each.

5. A rug 12 ft. by 17 ft. covers $\frac{1}{3}$ of the floor of a room 18 ft. wide. Find the length of the room.

PLAN. 1. Let L = the number of feet in length.
2. Then 18L = the number of feet in area.
3. Therefore, $\frac{1}{3}^{8}\mathrm{L} = 12 \times 17$. Why?
4. And L = —.

6. A rug 11 ft. by 15 ft. covers $\frac{2}{5}$ of the floor of a room 25 ft. long; find the breadth of the room.

7. A house 20 ft. by 52 ft. covers $\frac{2}{5}$ of a lot having 25 ft. frontage; find the depth of the lot.

8. Half the length of the Mississippi is 900 miles more than $\frac{1}{10}$ of its length; find its length.

Written.

1. Two brothers divided a farm of 160 acres so that one had 25 acres more than the other; how many acres had each?

2. The perimeter of a rectangle is 42 in. and the breadth is $\frac{2}{5}$ of the length; find the dimensions.

3. A picture requires 96 in. of molding to frame it. The length of the frame is $\frac{5}{3}$ of its breadth, outside measurement. Find the dimensions of the frame.

4. The breadth of a rectangular garden is $\frac{3}{4}$ of its length. The owner caused it to be covered 3 in. deep with black earth, costing him 60¢ a cubic yard. The expense was $15. Find the dimensions of the lot.

5. A rectangular park 100 yd. by 500 yd. is covered with lawns and flower beds; the lawn occupies 30,000 sq. yd. more than the flower beds; how many square yards are occupied by each?

6. One of the acute angles of a right-angled triangle is half the other; how many degrees are there in each?

7. One of the two equal sides of a triangle is $2\frac{1}{2}$ times the third side. The perimeter of the triangle is 21 ft. Find the lengths of the sides.

8. If a cubic foot of loose anthracite coal weighs 90 lb., how high must a bin 8 ft. by 10 ft. be to hold 30 tons?

9. Two boys measured the length of a playground. They took two sticks whose lengths they did not know, but saw that one stick was an inch longer than the other. One boy found the length of the playground to be 70 stick lengths, the other 68 stick lengths; what was the length of the playground? (Hint: Let x = length of shorter stick.)

10. The gauge of a water tank shows that it contains 620 gallons of water. The supply pipe furnishes 2 gallons per minute. At the end of 2 hours the gauge reads 560 gallons; what is the average consumption per minute?

If a body moves so that it passes over equal spaces in equal intervals of time, the body is said to move uniformly. In the following problems uniform motion is meant.

Written.

1. A man walked into the country at the rate of 3 mi. an hour. After spending $2\frac{1}{2}$ hr. there, he returned on the electric railway at the rate of 10 mi. an hour. His entire trip lasted 9 hours; how far did he go?

2. One wheel rider goes twice as fast as another. They start together and ride in the same direction about a circular track. In how many rounds will they pass for the first time?

PLAN. 1. Let r be the number of rounds made by the slower rider until they pass.
2. Then $2r$ = the number of rounds made by the faster rider.
3. But the faster rider must go one round more than the slower, in order to overtake him.
4. Therefore, $r + 1 =$ —. 5. Test.

3. Solve the same problem if the riders go in opposite directions.

4. Solve the same problem if one rider goes $1\frac{1}{2}$ times as fast as the other in the same direction.

5. Solve Exercise 4, the riders going in opposite directions.

6. A train with an average speed of 30 mi. per hour leaves Chicago at noon. At 2 P.M. a second train, with an average speed of 40 mi. per hour, leaves on the same line and in the same direction. When will the second train overtake the first?

7. A man set out to ride a distance of 54 miles. His horse trotted 6 mi. per hour. After he had gone a certain distance, his horse was lamed, and he had to proceed afoot, walking 3 mi. per hour. On arriving at his destination he found that he had ridden and walked an equal length of time. How many hours did it take him to make the journey?

Ratio. The quotient of two numbers is often called their *ratio.*

The questions: What part of 12 is 4? / What fraction of 12 is 4? / What is the quotient of $4 \div 12$? / What is the ratio of 4 to 12? } Are all answered by the fraction $\frac{1}{3}$.

Oral.

1. What is meant by the ratio of 4 to 2? Of 4 to 8?

2. Compare the ratios: $\frac{1}{2}$ and $\frac{2}{4}$; $\frac{3}{4}$ and $\frac{6}{8}$; and $\frac{1}{3}$ and $\frac{3}{9}$.

3. If $\frac{x}{8} = \frac{1}{2}$, what is the value of x? If $\frac{x}{8} = \frac{3}{9}$, what is the value of x? What is x, if $\frac{x}{2} = \frac{3}{5}$?

Proportion. The equality of two ratios is called a *proportion.*

Thus, $\frac{1}{2} = \frac{4}{8}$ and $\frac{x}{3} = \frac{3}{9}$ are proportions.

Proportional Numbers. Two or more pairs of numbers are said to be *proportional* when their ratios are equal.

Find the value of x *in each of the following proportions:*

4. $\frac{x}{3} = \frac{5}{9}$. 5. $\frac{x}{5} = \frac{12}{10}$. 6. $\frac{1}{6} = \frac{x}{7}$. 7. $\frac{2}{3} = \frac{x}{5}$.

Written.

8. An automobile traveled 70 mi. in 6 hours; how far did it travel in 15 hours at the same rate?

PLAN. 1. Let x be the number of miles traveled in 15 hours.
2. Then $\frac{x}{15}$ is the rate per hr., but $\frac{70}{6}$ is also the rate per hr.
3. Therefore, $\frac{x}{15} = \frac{70}{6}$. Why? Therefore, $x =$ —.

9. If a sum of money earns \$48 interest in 5 years, how much will it earn in 16 years at the same rate per cent?

10. A city whose population was 40,000 had 2,500 children of school age; the total population increased to 48,000, and the number of children of school age increased proportionally. How many children of school age were there then?

11. A man with an income of \$800 saved \$60. The next year his income was \$1,000 and he saved a proportional amount. How much did he save the second year?

Besides problems solved by unitary analysis, page 133, and the problems of simple proportion, as on the previous page, there are problems that involve the **principle of partition.**

Partitive Proportion. The method used to divide a quantity into parts proportional to several given numbers is called *partitive proportion* and will be understood best from an example.

EXAMPLE: An inheritance of $8,000 was divided among 3 heirs in proportion to their ages: 5 yr., 15 yr., and 20 yr.; how many dollars did each receive?

SOLUTION. 1. The statement means that each receives the same sum for each year of his age.
2. Let x = the sum received for 1 year.
3. Then the first receives $5x$, the second $15x$, and the third $20x$.
4. Altogether they receive $5x + 15x + 20x = 40x$.
5. Hence $40x = 8{,}000$.
6. Then, $x = 200$, $5x = 1{,}000$, the share of the first, etc.

Written.

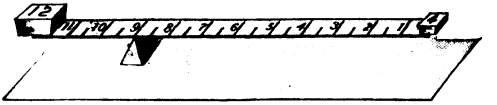
1. A invests $3,000, B $8,000, and C $11,000 in a factory. The profits for the year are $2,000, shared in proportion to the amount of capital invested; what did each receive? (Let p = the profit on $1,000. Then, A receives $3p$, etc.)

2. A street was paved at a cost of $2,400. The cost was assessed on the property owners in proportion to the frontage owned. Mr. A owned 100 ft., Mr. B 300 ft., Mr. C 150 ft., Mr. D 650 ft.; how much did each pay?

3. Three men rented an automobile for $140 per month of 30 days; one used it 1 day each week, another 2 days, and the third 4 days; how much of the monthly rental should each pay?

4. Air is composed of 21 volumes of oxygen to 79 volumes of nitrogen; how many cubic inches of each are there in a cubic foot of air?

Experiment. Alfred placed a stick under a foot-rule at the 9-inch mark. He then placed a 12-ounce block on the short end and a 4-ounce weight on the other and found that the objects balanced. By placing the support under the rule at different places he balanced the block in each of the following cases:



Support at	Distance from support to end carrying block.	Weight of block.	Dist. from support to end carrying weight.	Weight to balance block.
9 in.	3	12 oz.	9	4
8 in.	4	" "	8	6
6 in.	6	" "	6	12
4 in.	8	" "	4	24
3 in.	9	" "	3	36

Oral.

1. When the support was in the center of the rule, what was the weight required to balance the block?

2. When the support was between the center and the end bearing the block, how did the weight used compare with the weight of the block?

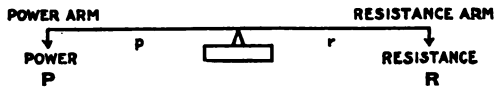
3. When the support was between the center and the end bearing the weight, how did the weight used compare with the weight of the block?

4. In the first line of the table compare the product of the numbers in columns 2 and 3 with that of the numbers in columns 4 and 5. What do you find?

5. Make a similar comparison for each line of the table. What do you find throughout?

6. If convenient, the pupils should make a similar table, using a rule of different length, and answer the questions above with respect to their own results also.

Terms Used. In a lever the arm to which the power is applied is called the *power arm; p* denotes the length of this arm. The arm which acts on the resistance is called the *resistance arm; r* denotes the length of this arm. P stands for power and R for resistance.



The Law of Levers. When the power just balances the resistance,

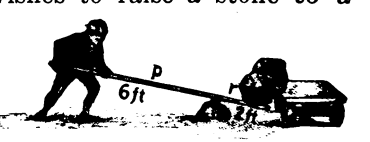
$$Pp = Rr.$$

In words: **The number of units of weight in the power times the number of units of length in the power arm equals the number of units of weight in the resistance times the number of units of length in the resistance arm.** This is called the **Law of Levers.**

More briefly: **The power times the power arm equals the resistance times the resistance arm.**

The power and the resistance must be measured in the same unit (as ounces, pounds, or tons) and the lengths of the two arms in the same unit (as feet or yards).

1. A boy weighing 75 lb. wishes to raise a stone to a truck. He uses a lever with arms 6 ft. and 2 ft.; how heavy a stone can he raise by throwing his whole weight on the bar?



PLAN. 1. Let W be the weight of the stone in pounds.
2. Then, $2 \times W = 6 \times 75$ lb. 3. Therefore, W = — lb.

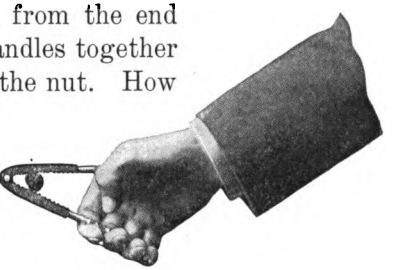
2. What is the length of the resistance arm of a lever when the power arm is 3 ft. long, the power 15 lb., and the resistance overcome 75 lb.? Express the solution in the form of an equation.

3. The power arm of a lever is 10 ft. long and the resistance arm 2 ft. long; how much power is needed to overcome a resistance of 1,000 lb.? Of $1\frac{1}{2}$ tons?

By use of the law of levers find the missing numbers in the table:

	POWER.	POWER ARM.	RESISTANCE.	RESISTANCE ARM.
1.	20 oz.	3 in.	3 oz.	——
2.	125 lb.	——	625 lb.	2 ft.
3.	——	18 ft.	600 lb.	3 ft.
4.	100 lb.	9 ft.	——	2 ft.
5.	¼ ton	——	5 tons	18 in.

6. Frank noticed that a nut cracker is a lever with the fixed point at the end. The length of the handles were 6 in. and the distance of the nut from the end was 1 in. He pressed the handles together with a force of 5 lb. to crack the nut. How heavy a weight, if placed directly on the nut, would be required to crack it?



7. What pressure must be exerted on the handles of the nut cracker of Example **6** to crush a nut requiring 75 lb. direct pressure? To crush one requiring 165 lb.?

8. Ralph exerted a pressure of 1 lb. in closing the handles of a pair of scissors. According to the dimensions given in the picture, with what force were the blades pressed together in cutting an object at the point?

9. What pressure would be exerted at a point on the blades 2 in. from the rivet? When an object is hard to cut, where do you place it on the blade? Why?

10. Two men, A, B, carry a weight of 200 lb. on a 6-foot bar resting on a shoulder of each. The weight hangs 2½ ft. from A. Let x represent the number of pounds carried by A. What will represent the amount carried by B? Apply the equation and find how many pounds each carries.

1. Roy observed that in June 18 more days were fair than cloudy and that it rained on $\frac{1}{2}$ of the cloudy days; how many days were rainy? Cloudy? Fair?

2. Sea water is 1.026 times as heavy as fresh water and contains $\frac{1}{14}$ by weight of salt. Fresh water weighs $62\frac{1}{2}$ lb. a cubic foot; how many cubic feet of sea water must be evaporated to obtain 1 ton of salt?

3. Experiments have shown that a cast-steel wire .04 in. in diameter can support 176 lb., but breaks under any greater weight. The weight of the steel being 4.5 oz. a cubic inch, how long a piece of this wire suspended at the upper end can hang freely without breaking?

4. A certain variety of stone weighing 2.5 oz. per cubic inch can support a weight of 3,600 lb. per square inch, but is crushed under any greater weight. Accepting as a rule of construction that not more than $\frac{1}{15}$ of the crushing weight should be permanently put upon a stone, how high a wall may be built of this stone?

5. In 3 gallons of fruit jelly there were 6 pints more of fruit juice than of sugar; how many pints of fruit juice were there in the whole amount?

6. A man spends $3x\%$ of his income for food, $x\%$ for rent, $2x\%$ for clothing and $\frac{1}{2}x\%$ for miscellaneous expenses, and saves $3\frac{1}{2}x\%$; find x. If his income is \$1,500, find his expenditures for each item in dollars?

7. A man spends 50% of his annual income for food and rent, 40% for other expenses and has left \$12 more than 10% of what he spent. Find his income.

8. In a factory there are three rates of pay, \$1.25, \$1.50 and \$2.00 a day. There are 20 more men receiving \$1.50 than \$1.25, and 48 more receiving \$2.00 than \$1.50. The daily pay-roll amounts to \$451; how many men are working at each rate?

Oral.

When l = length, b = breadth, a = area, and p = perimeter of a rectangle, state the meaning of:

1. $a = lb$. 2. $\frac{a}{l} = b$. 3. $\frac{a}{b} = l$.

4. $p = 2b + 2l$. 5. $p - 2l = 2b$. 6. $\frac{p}{2} - b = l$.

7. What represents the interest on P dollars for 1 year at 6%? For 3 years? For t years?

8. When x is the rate of motion of a train per hour, y the number of hours traveled, and d the distance, state the meaning of: $d = xy$; $\frac{d}{y} = x$; $2d = 2xy$; $\frac{1}{2}xy = \frac{1}{2}d$.

Written.

9. Divide the sum \$5,000 into two parts so that the interest on the larger at 3% a year shall equal that on the smaller at 5% a year.

10. Mr. Gray sold his house and lot for \$17,670. He invested part of the proceeds at $5\frac{1}{2}$% and the remainder at $4\frac{1}{2}$%. The income from each part was the same; how was the sum divided?

11. The sum \$15,000 is invested, part at $4\frac{1}{2}$% and the rest at 5%. The annual income is \$687; how is the investment divided?

12. A man bought a tract of land at \$75 an acre. He sold $\frac{1}{3}$ of it at \$84; $\frac{1}{4}$ at \$85, and the remainder at \$90 an acre, thus gaining \$11,280; how many acres were there in the tract? What per cent of the purchase price did he gain?

13. Two persons save \$300 apiece annually. The first had already saved \$1,500 when the second began. In how many years after the second began will he have half as much as the first then has?

14. A bookseller bought 656 copies of a book at a discount of 25%. He sold half of them at list price and the other half in larger quantities at a discount of 10%. He made \$209.92; find the list price.

XXII

POWERS AND ROOTS

SQUARES

Oral.

1. What is the area of a square 4 in. on a side? 5 in.? 3 in.? 9 in.? 8 ft.? 10 yd.?

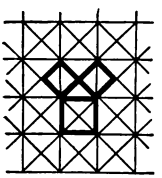


Fig. 1.

2. What is the size of the largest angle in each little triangle in the figure? What kind of triangles are they? How do they compare with one another in size?

3. If one triangle is singled out, there is a square on each side, as shown by the heavy lines in Figure 1. How many triangles are there in one of the small squares that border the triangle? In both the small squares? In the large square?

4. Compare the sum of the areas of the small squares with the area of the large square.

Hypotenuse. The side opposite the right angle of a right triangle is called the *hypotenuse.*

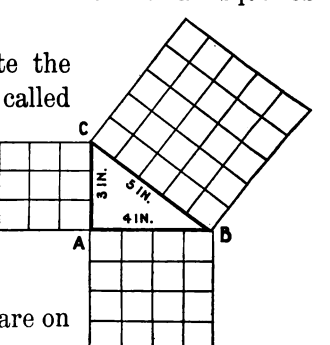


Fig. 2.

5. What is the length of each side of the triangle in Figure 2?

6. What is the area of the square drawn on the hypotenuse?

7. Compare the area of the square on the hypotenuse with the sum of the areas of the squares on the other two sides.

8. Construct a right-angled triangle having a hypotenuse 10 in. and one side 6 in. Find the area of the square on each side of this triangle. Compare as in Exercise 7.

Construct right triangles having the following sides including the right angle:

1. 9 in., 12 in. 2. 5 in., 12 in. 3. 12 in., 16 in.

4. Find the areas of the squares on the three sides of each triangle in Exercises 1, 2, 3.

5. Compare the square on the hypotenuse with the sum of the squares on the other two sides.

6. Figure 1 is a square in which T is a right triangle and A and B are squares. Each triangle is equal to T.

Figure 2 is a square equal to Figure 1 and containing the same triangles. C is a square.

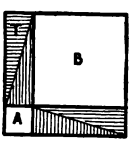


Fig. 1.

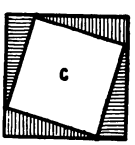


Fig. 2.

Think of the triangles as taken from each figure; how do A plus B compare with C? If convenient, cut such figures out of paper and verify the relations.

In any right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

If h represents the length of the hypotenuse and a and b those of the sides then $h^2 = a^2 + b^2$.

This relation is called the *Pythagorean Theorem*, after the Greek philosopher and mathematician, Pythagoras (about 500 B.C.), who is supposed to have discovered it.

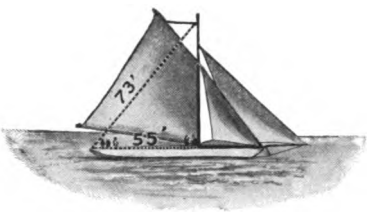
7. The hypotenuse of a right triangle is 10 in.; find the area of the square having this line for a side. The area of the square on one of the other sides is 36 sq. in.; what is the area of the square on the third side of the triangle? How long is a side of this square? The third side of the triangle?

Find the unknown sides of the following right triangles:

	8.	9.	10.	11.	12.
Hypotenuse:	5	x	13	10	x
Side:	4	3	x	6	6
Side:	x	4	12	x	8

NOTE.—The square roots to be extracted on this page may all be found by inspection, or by the method of factoring explained on page 190.

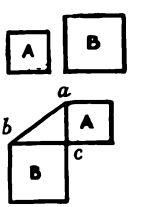
1. A rope 73 ft. long is fastened to the top of a boat's mast, and when drawn taut, it touches a point on the deck 55 ft. from the foot of the mast. Find the height of the mast.



2. A garden has the form of a right triangle. Its hypotenuse is 149 ft. and one side 51 ft.; what will it cost to surround it with a fence at \$.70 a running foot?

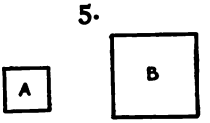
3. If a ladder 41 ft. long rests against a wall with its foot 9 ft. from the wall, how high is the top of the ladder from the ground?

4. To find a square whose area is equal to the sum of the areas of two given squares. A and B are the given squares. Place them as shown in the figure. What kind of a triangle is *abc*? Why? How may the required square be constructed?

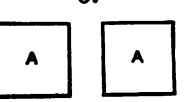


Construct a square equal in area to each pair of given squares:

5.



6.



7. Draw any two squares. Construct a square equal in area to the sum of the squares.

Extract the square roots by factoring:

8. $\sqrt{625}$	9. $\sqrt{169}$	10. $\sqrt{361}$	11. $\sqrt{256}$
12. $\sqrt{441}$	13. $\sqrt{1024}$	14. $\sqrt{8464}$	15. $\sqrt{4225}$
16. $\sqrt{1225}$	17. $\sqrt{3025}$	18. $\sqrt{7744}$	19. $\sqrt{5625}$

Extraction of the Square Root of Any Number. The methods so far used for finding square root are sufficient for the easier cases; the following method is more general:

1. *Point off the number into periods of two figures each, beginning at the right (at the decimal point in a decimal).*

2. *By inspection find the largest integer whose square is not greater than the left period.* (In Example A it is 9.)

3. *Use this integer as the first digit of the root. Subtract its square from the left period.* (In Example A this square is 81.)

4. *Bring down the next period.* (In Example A this makes 364.)

5. *Multiply the part of the root already found by 2. This number is called the* **trial divisor.** (18 in Example A.)

6. *Divide the remainder (omitting the right digit) by the trial divisor and use the digit found as the next digit of the root.* (In Example A, $36 \div 18 = 2$.)

7. *Annex this digit to the trial divisor. This forms the* **complete divisor.** (182 in A.)

8. *Multiply the complete divisor by the digit of the root just found and subtract.*

NOTE.—It may happen that the product to be subtracted is larger than the number from which it is to be subtracted. This indicates that the trial divisor led to too large a digit. Try the next smaller digit.

9. *Repeat the steps 4 to 8 until all the periods have been brought down.*

If the last remainder is zero, as in Example B, the process is ended, the given number is a perfect square, and its root has been found exactly. If the last remainder is not zero, as in (C), the process may be continued as far as desired by supplying zeros.

Test. The square of the root = the given number.

NOTE.—The reasons for the process above are too difficult to be understood easily at this stage.

(A)

Root	9 2
Number	84′64
	81
18	364
182	364

(B)

Root	3 0. 6 9
Number	9′41′.87′61
	9
6	41
60	4187
606	3636
612	55161
6129	55161

(C)

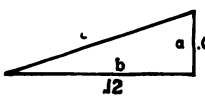
Root	1 4. 1 4+
Number	2′00.
	1
2	100
24	96
28	400
281	281
282	11900
2824	11296
	604

Square Root of Fractions. Either reduce the fraction to a decimal or extract the square root of both terms.

Written.

Extract the square root of each number to two decimal places:

1. 361. 2. 784. 3. 1,681. 4. 1,936.
5. 841. 6. 2,025. 7. 300. 8. 5.
9. $\frac{16}{25}$. 10. $\frac{49}{144}$. 11. $\frac{1}{5}$. 12. $\frac{2}{3}$.



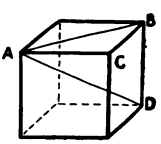
13. Find the length of c, according to the figure. Also, find a, if c were .17 in. and b were .08 in.

Find the other side of the following right-triangles; if there is a decimal part, carry the value to one decimal place:

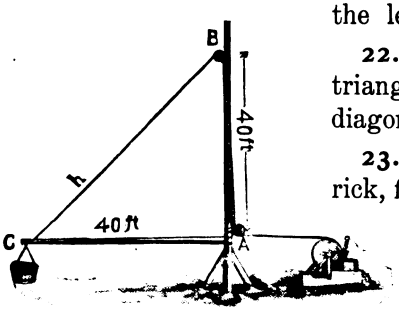
	14.	15.	16.	17.	18.
Hypotenuse:	205	——	221	227	200
Side:	133	187	140	——	150
Side:	——	84	——	115	——

19. Measure the length and width of a book cover and compute the diagonal to .1 in. Test the result by measurement.

20. Find the length to .01 ft. of a diagonal of a square 1 ft. on a side.

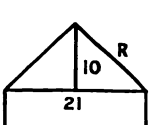


21. This is the picture of a cube 1 ft. on an edge. Using the result of Exercise 20, state the length of AB.



22. Triangle ABD is a right-triangle; find the length of the diagonal AD to .01 ft.

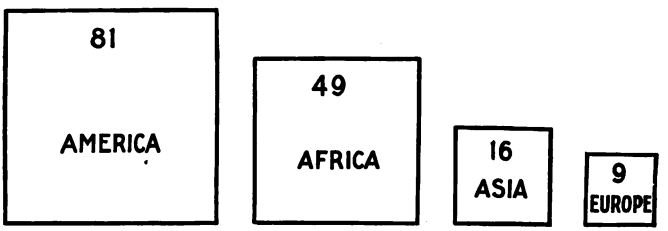
23. In the picture of the derrick, find the length of the rope, h, to the nearest foot; also of the rope from A to B to C.



24. Find the length of the rafter R used for this roof.

The approximate number of acres to each inhabitant in various parts of the world is shown in the following diagram:

For example, there are 81 acres of American territory to each inhabitant of America.



Oral.

1. What is the square root of the number in each square?

2. Measure a side of each square. Taking $\frac{1}{8}$ in. as the unit, what number represents the side of each square? Are the squares correctly drawn?

Written.

3. China has approximately a population of 350 millions, British India 278 millions, and Japan 40 millions. Extract to the nearest unit the square roots of 350, 278, and 40, and construct squares to represent these populations.

4. Find to the nearest mile the length of the sides of squares having the same area as:

State.	Area in sq. mi.	State.	Area in sq. mi.
Connecticut	4,990	New Jersey	7,815
Delaware	2,050	Rhode Island	1,250

Using $\frac{1}{8}$ in. to represent 1 mi. draw lines to represent the lengths found; then draw squares to represent the areas.

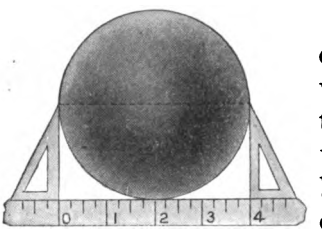
5. In 1900, 26,120,000 of the inhabitants of the United States were under 15 years of age; 44,800,000 were from 15 to 59 years of age, and 4,870 were 60 years of age or over. By extracting the square roots of 2,612, 4,480, and 487 to the nearest unit, find the lengths of the sides of squares to represent these numbers. Draw the squares.

XXIII

FORM STUDY AND MEASUREMENT

VOLUMES OF SPHERES

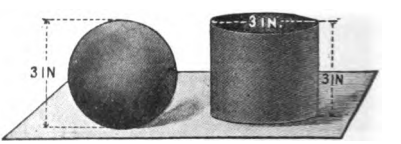
1. Henry measured the diameter of a croquet ball and found it to be 4 in. Explain from the figure how it was done.



2. He filled a rectangular dish of base 11 in. by 16 in. with water to a depth of 5 in. When the ball was immersed in the water the depth was found to be $5\frac{4}{21}$ in.; what was the volume of the ball?

3. If convenient, perform a similar experiment.

4. Louise modeled from clay a cylinder and a sphere with the dimensions given. The clay cylinder weighed 30 oz. and the sphere 20 oz.; the weight of the sphere was what part of that of the cylinder?



5. If convenient, perform a similar experiment.

6. Find the volume of a cylinder whose base has a diameter $2r$ and whose altitude is $2r$.

7. According to Exercise **4**, what part of this volume is the volume of a sphere of diameter $2r$? What represents the volume of the sphere?

The volume of any sphere of radius r is $\frac{4}{3}\pi r^3$.

8. What is the value of $\frac{4}{3}\pi r^3$, taking $r = 8$ and $\pi = 3.1416$?

9. What is the volume of a sphere whose radius is 7 in.?

10. Find the volume of a sphere of radius 3 ft.; 1 yd.; 2 in.; 4 in.; 5 ft.

For any sphere let d = diameter, a = area, r = radius, v = volume

Written.

Fill out the blanks:

	1.	2.	3.	4.	5.	6.	7.	8.
$r =$	—	2	—	—	$\frac{1}{3}$	—	$1\frac{2}{10}$	—
$d =$	—	—	—	8	—	$\frac{4}{7}$	—	$\frac{6}{10}$
$a =$	4π	—	36π	—	—	—	—	—
$v =$	—	—	—	—	—	—	—	—

9. Regarding the earth as a sphere of radius 4,000 miles, find its volume. (Use $\frac{22}{7}$ as an approximate value for π.)

10. The gilding of a ball on a church spire cost \$126.72 at \$1.12 per square foot. Find the diameter of the ball.

11. The inner surface of the dome of the capitol at Washington is covered by a fresco painting. Regarding the surface as a hemisphere 165 ft. in diameter, how many square feet are there in the painting?

12. A balloon has the form of a sphere 10 yd. in diameter; how many square yards of material are needed to make the balloon?

13. How many cubic feet of gas are required to inflate the balloon?

14. What is the cost of inflating the balloon with gas costing \$1.40 per 1,000 cu. ft.?

15. A kettle is in the form of a hemisphere 28 in. in diameter; how many cubic inches of water does it hold?

16. On a church spire there is a ball 2 ft. in diameter; what will it cost to gild the ball at \$1.20 per square foot?

17. What represents the volume of a hemisphere of radius r? What represents the volume of $\frac{1}{8}$ of a sphere of radius r?

Oral.

1. An object floating in water displaces its own weight of water. If a block weighing 1 lb. floats in water, what is the weight of the water displaced?

2. If a ball floating in water is half immersed, its volume is how many times the volume of the water displaced?

3. If the volume of the ball of Exercise 2 is 1 cu. in., how much water does it displace?

4. Which is the heavier per cubic inch, water or the ball? How many times as heavy? The ball is how many times as heavy as water per cubic inch?

5. If a piece of wood when floating is $\frac{1}{3}$ under water, what is the ratio of the weight of the wood to that of an equal bulk of water?

6. If a floating piece of cork is $\frac{1}{6}$ under water, what is the ratio of the weight of the cork to that of an equal bulk of water?

7. What is the similar ratio of the weight of iron to that of water, if a cubic foot of water weighs 1,000 oz. and a cubic foot of iron 7,500 oz.?

Specific Gravity. The ratio of the weight of any substance to the weight of an equal volume of water is called its *specific gravity.*

8. 1 cu. ft. of water weighs 1,000 oz. If one cubic foot of marble weighs 2,700 oz., what is its specific gravity?

Written.

9. The specific gravity of gold is 19.3; what is the weight in pounds of a cubic foot of gold?

10. Find the weight of one cubic foot of each substance whose specific gravity is given:

Lead	11.3	Cork	0.24	Granite	2.7
Nickel	8.9	Silver	10.5	Mercury	13.59
Sulphur	2.0	Copper	8.9	Milk	1.03

Written.

1. If cast iron weighs 7.5 times as much as water and if one cubic foot of water weighs $62\frac{1}{2}$ lb., find the weight of a cast-iron cannon ball 1 foot in diameter.

2. What is the weight of a 2-inch ball of ivory, if the specific gravity of ivory is 1.9?

3. How many $\frac{1}{2}$-inch lead bullets weigh a pound, if the specific gravity of lead is 11.3?

4. A marble monument consists of a sphere 2 ft. in diameter resting on a cylindrical column 18 in. in diameter and 5 ft. high. What is the weight of the monument, if the specific gravity of marble is 2.7?

5. The roof of a mosque is in the form of a hemisphere 80 ft. in diameter; what will it cost to paint it at 70¢ per square yard?

6. A kettle in the form of a hemisphere is 12 in. deep inside. Calculate its contents in cubic inches. Find the area of its outer surface if the walls of the kettle are everywhere $\frac{3}{4}$ of an inch thick.

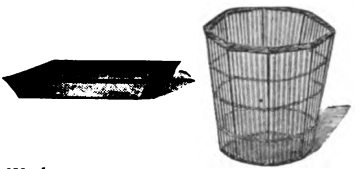
7. A silver sphere 22 in. in outer circumference has its walls $\frac{1}{4}$ in. thick. Find the weight of the sphere, if the specific gravity of silver is 10.5. Find the value of the silver at 90¢ per ounce.

8. If the earth be regarded as a sphere of radius 4,000 miles, what is its area?

9. A spherical balloon of radius 5 yd. is inflated with hydrogen. If a cubic yard of air weighs 2.2 lb. and a cubic yard of hydrogen .15 lb., what is the upward pull of the balloon, that is, the difference between the weight of the hydrogen and the weight of the air displaced?

10. How many cu. in. in an orange $2\frac{1}{2}$ in. in diameter? In one 4 in. in diameter? Which is more advantageous, to buy the large size at 5¢ each or the small size at $2\frac{1}{2}$¢ each?

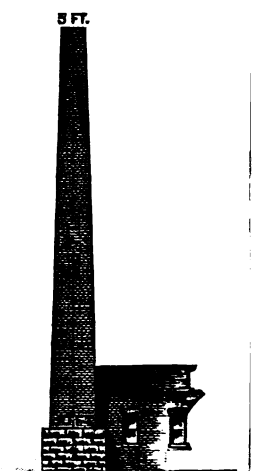
Frustums of Pyramids. When a pyramid is cut parallel to the base a smaller pyramid is cut off. What is left is called a *frustum of a pyramid.*



Frustums are seen in such common forms as are shown in the figures.

Written.

1. The chimney in the figure is a frustum of a square pyramid with the dimensions marked; what is the area of each base? What kind of figures are the lateral faces? Find the area of one of them; of all of them. What is the total surface of the frustum?



2. A tin pan measures 8 in. by 12 in. on the bottom and 10 in. by 15 in. on the top. The slant height is 6 in.; how many square feet of tin are required to make it?

3. A waste basket has regular hexagons as bases and measures 54 in. around the bottom and 72 in. around the top. The slant height is 17 in.; how many yards of silk 24 in. wide are required to line its inner sides, excluding the bottom, and allowing $\frac{1}{2}$ yd. for waste?

4. A manufacturer wishes to purchase cardboard to make 10,000 lamp-shades. The shades are frustums of regular hexagonal pyramids and measure 15 in. around the top and 36 in. around the bottom with a slant height of 6 in. He buys cardboard 19 in. by 24 in. by the ream of 480 sheets and finds that 10% of each sheet is wasted in cutting; how many reams must he buy?

Frustums of Cones. By cutting a cone parallel to the base a smaller cone is cut off. What is left is called a *frustum of a cone.*

If C = convex surface, R = radius of lower base,
S = slant height, r = radius of upper base,
A = altitude, V = volume,

it can be proved that:

1. $C = (R + r)\,\pi\,S$.

2. $V = \dfrac{\pi A (R^2 + r^2 + Rr)}{3}$.

1. State these formulas in words.

Their proof is too difficult to be undertaken here, but they may be used in solving the following exercises. Use $\frac{22}{7}$ for π and simplify by canceling as much as possible.

Written.

2. The bottom of a dish pan is 10 in. in diameter, the top is 14 in. in diameter and the slant height of the pan 9 in.; how many square inches of tin are required to make it?

3. A tub is 26 in. in diameter at the bottom, 40 in. at the top, and 24 in. high, inside measure; how many gallons does it hold? (231 cu. in. = 1 gallon.)

4. The slant height of the same tub is 25 in., inside measure; how many square feet of lumber are there in it, disregarding the thickness of the lumber?

5. A lighthouse tower is in the shape of a frustum of a cone 20 ft. in diameter at the base, $10\frac{1}{2}$ ft. at the top, and 45 ft. high. The hollow portion within is a cylinder 8 ft. in diameter; how many cu. ft. of masonry in the tower?

6. The slant height of the same tower is 45 ft. 3 in.; what will it cost to paint the walls outside and inside at $3\frac{1}{8}¢$ a square foot?

7. The tank of a street sprinkler is in the form of a frustum of a cone with inside diameters 8 ft. at the bottom, 6 ft. at the top, and inside height 5 ft.; find its capacity in cubic feet.

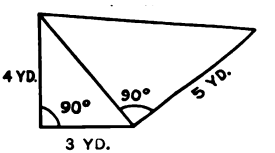
Written.

Find.

1. $\sqrt{625}$ 2. $\sqrt{324}$ 3. $\sqrt{441}$ 4. $\sqrt{93025}$ 5. $\sqrt{102400}$

Find the unknown parts of the right triangles:

	SIDES.	HYPOTENUSE.	AREA.
6.	5	13	y
7.	3	y	6

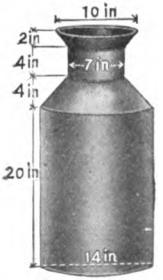


8. A piece of land has the dimensions indicated. Find its area.

9. If the hypotenuse of a right triangle is $41q$ and one side is $40q$, what is the other side?

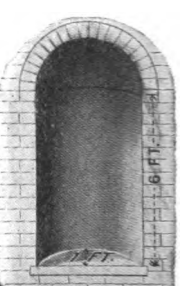
10. If the area of a right triangle is $600a^2$ and the base is $30a$, what is the hypotenuse?

11. A trunk measures 18 in. by 32 in., inside measure; what is the longest umbrella that can be laid flat on the bottom of the trunk? State the result to .1 in.



12. A milk can has the dimensions indicated; how many square inches of tin are required to make it?

13. At \$.85 a square foot, how much will it cost to paint a conical spire, the diameter of the base being 20 ft. and the height of the spire 40 ft.?



14. The niche shown in the figure has a semicircular base and the top is in the form of a quarter sphere; what is the cost of gilding the niche at 42¢ a square foot?

15. The equatorial radius of the earth is 3,963.25 mi.; what is the length of the equator?

16. The diameter of the moon is about 1,080 miles; what is the length of its equator?

XXIV

GENERAL REVIEW

PROBLEMS

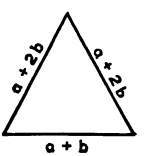
1. What is the cost of 12 chairs at $\$1.12\frac{1}{2}$ each?

2. What is the cost of 8 books at $\$.87\frac{1}{2}$ each?

3. How many flags at \$15 each can be bought for \$285?

4. How many wagons at $\$87\frac{1}{2}$ each can be bought for \$2,450?

5. Find the perimeter of this triangle.



6. By how much does the sum of the sides exceed the base?

7. Add: $a + 3b + c$ and $6b + 8c$

8. Subtract: $10c + \frac{3}{2}d$ from $5c + \frac{1}{2}d$

9. In a certain year the ten leading butter states of the United States produced the following amounts:

State.	Million pounds.	State.	Million pounds.
Iowa	139	Illinois	87
New York	115	Minnesota	82
Pennsylvania	111	Michigan	68
Wisconsin	107	Kansas	60
Ohio	88	Indiana	55

Represent these amounts graphically to the nearest 10 million, using parallel lines.

10. In a recent year 99,311,678 lb. of crude rubber were imported into the United States; the average price was \$4.47 per pound; what was it worth?

11. Spain yields 32,800 tons of cork annually, valued at \$183 a ton; what is the yield worth?

1. Find the amount of the following bill:

Sept. 6	1 enameled dish pan.	75	Sept. 8	1 ½-inch bit.....	45
	1 " basin....	20		1 pt. turpentine.....	10
	1 8-quart pan.....	14	" 10	oil and turpentine...	10
	2 oz. putty.....	08	" 12	1 steamer.....	25
Sept. 8	1 saw.....	90		1 screw-driver.....	18
	1 wrench.....	45		1 flour sieve.....	20
	2 oz. 2-inch brads ...	12	" 15	2 asbestos mats	10
	2 doz. wardrobe hooks	20	" 17	1 letter plate.....	50
	1 file.....	10		3 papers tacks.....	15
	1 pair 8-inch pliers ..	60	" 21	1 chopping knife....	25
	1 brace.....	75	" 25	4 Welsbach gas lamps	4.00
	1 plane.....	1.25	" 27	1 box tapers	10
	1 ¼-inch bit.....	25		1 gas lighter.....	25

2. 1,000,000 bales of cotton often change owners on the New York Exchange in one day during the winter season; what is the value of such a sale at 12¢ a pound, allowing 500 lb. to the bale?

3. A cotton gin cleans 1,000 lb. of cotton in the time it would take a man to clean 5 lb.; the cotton gin does how many times the work of one man?

4. Find the total number of bushels of wheat received in the markets of these cities on the date of the statement. On the same date a year before.

5. If the wheat was sold at an average price of $.75 a bushel, what was its value in each case?

The receipts of wheat were as follows:

	To-day. Bushels.	A year ago. Bushels.
Chicago.....	15,000	136,208
Milwaukee.....	6,160	880
Minneapolis	71,200	132,050
Duluth.....	6,953	17,260
St. Louis.....	23,000	28,000
Kansas City.....	18,900	36,000

6. What was the brokerage for selling each amount at ⅛%?

1. At 5 per cent per annum, what is the annual interest on \$2,000? On A dollars? What is the interest on the latter amount in 4 years? In n years?

2. At n per cent interest per annum, what is the annual interest on D dollars? What is the interest for n years?

3. 35% of a certain capital is invested at 4% interest, 45% at 5%, and the remainder at 6%. The annual income from the whole is \$15,132; find the capital and the amount invested at each rate.

4. A commercial traveler receives a commission of 5% on his sales. On Tuesday his sales amounted to 3 times as much as on Monday. His commission for the two days was \$24; how much were his sales for each day?

5. During a certain year a man earns $\frac{1}{3}$ as much as he has already saved. He spends \$1,000 in the year and increases his savings by $\frac{1}{5}$ of their amount; how much does he save?

6. Four workmen receive together \$560 for a job on which they worked 30, 24, 20, and 17 days respectively; how much should each one receive?

7. There were made in the United States in one year four times as many pairs of rubbers for women as for children, 5 million pairs fewer for men than for women, and altogether 3,000,000 pairs more than 7 times the number for children; find the number of each kind made.

8. In both houses of Congress there were recently 480 members. If the number in each house had been increased by 10, the number of Representatives would have been 4 times the number of Senators; find the number of each.

9. Missouri sent 6 fewer Representatives to Congress than Ohio and 2 more than Texas. If the delegation of Missouri had been increased by 1, it would have been $\frac{2}{3}$ of that of Ohio; find the number in each delegation.

1. Mr. Roe sold 5 shares of a carpet company's stock at $110 a share; how much was this above the par value of the stock? How much did he gain on 5 shares bought at par?

2. An investor in the stock of a cement company sold 10 100-dollar shares at $90 each; how much below par was this? If he bought them at par how much did he lose?

3. A broker sold 150 shares of copper stocks at $125 a share and charged $\frac{1}{8}\%$ brokerage; how much did the seller receive?

4. What did 75 shares of railroad stock at $110 per share cost Mr. Gray (brokerage, $\frac{1}{8}\%$)?

5. Mr. White's broker sold 25 shares of railroad stock for him at $116 a share, $\frac{1}{8}\%$ brokerage; what did Mr. White receive?

6. A man bought 50 shares of Pullman Palace Car stock at 220. If an annual dividend of 8% is paid, how much does he receive a year? What per cent of the amount invested is this?

7. In Galveston, Texas, 11,000,000 cu. yd. of earth were used to build an embankment, costing $18\frac{1}{2}¢$ per cu. yd. To pay for the improvement the city issued $1,000 bonds; how many were required?

8. When goods listed at $45 are purchased at a discount of $\frac{1}{3}$, and 20%, what is their cost?

9. A collector was to receive 2% of the first $5,000 of his receipts and $1\frac{1}{2}\%$ of the remainder of his receipts. He turned in $8,934.20; how much did he collect?

10. In a leap year how many days are there from Washington's birthday to Memorial Day?

11. When the time after noon is $\frac{2}{3}$ of the time to midnight, what time of day is it?

1. Twice the recent population of Boston increased by 880,000 equals four times the population diminished by 240,000; what was the population of Boston?

2. A man leaves $11,000 to his wife, 2 sons and 3 daughters. According to his will, each daughter receives the same amount; each son receives twice as much as a daughter and the wife twice as much as a son; how much does each receive?

3. A merchant had in stock 1,169 yd. of silk. He sold a certain number of yards to a first customer, twice as many to a second, half as many to a third and four times as many to a fourth customer, and had 1,014 yards left; how many yards did he sell to each?

4. At a certain election there were two candidates; 51,149 votes were cast and the candidate elected had a majority of 259; how many votes did he receive? How many did the other receive?

5. The sum of two numbers is 384; one of them is 18 greater than the other; find the numbers.

6. In 1900 the population of Pennsylvania exceeded that of Illinois by 1,480,565. Together their population was 11,123,665; find the population of each.

7. It is 93 million miles from the earth to the sun. Regarding the path as circular, about how many miles does the earth travel in a year? In a day? In an hour? In a second?

8. The volume of a cone is what part of that of a cylinder having the same base and altitude?

9. A cylinder of iron weighed 852 lb. Without changing its length, it was tapered from the middle toward one end into the shape of a cone. Find the weight of the resulting bar.

1. The circulation of money per capita in this country recently was $29. Taking the population to be 76,000,000, how many dollars were in circulation?

2. If a commission merchant returns $95 to a customer after having deducted 5%, what is the amount of his sale?

3. If an agent returns $133 to a customer after having deducted 5%, what is the amount of the sale?

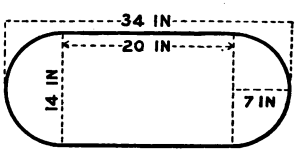
4. When the rate of commission is 4%, the commission on a sale is $16, what is the amount of the sale?

5. If the commission for a purchase at the rate of 6% is $18.60, what is the amount of the purchase?

6. A woman inherited $5,000. How many shares of United States Express Company stock at 114 could she buy? (Fractions of a share can not be bought.) How much would she have left?

7. A steam gauge indicates the pressure on a square inch. When the steam gauge reads 45 lb. it means that the steam pressure is 45 lb. on every square inch of inner surface of the boiler. At this rate what is the entire pressure in tons on a cylindrical boiler 2 ft. in diameter and 8 ft. long?

8. A wash boiler is 16 in. deep and has the base indicated in the figure; what is its capacity?

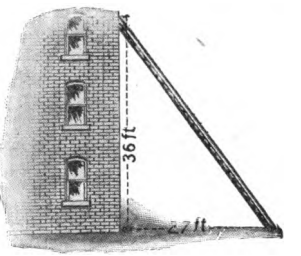


9. What is the radius of a circle whose area is 3.14159 square feet?

10. It is estimated that 15,000,000 cu. ft. of water pass over Niagara Falls a minute. If the water which passes over in one hour were frozen without change of volume into a cylindrical icicle of the largest diameter which would permit it to be set up on a playground 200 ft. square, how high would the icicle be to the nearest mile?

1. A garden in the form of a right triangle has a hypotenuse 50 ft. long, and one side 30 ft. long; at \$.70 a running foot, what is the cost of a fence enclosing the garden?

2. A ladder is placed against a building, as shown in the figure; how long is the ladder?



3. The foot of a ladder 30 ft. long is set 6 ft. from the base of a building; how high does it reach vertically on the building?

The following list gives the dimensions and cost of the principal buildings of the World's Fair in St. Louis in 1904:

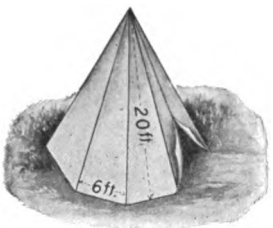
BUILDINGS.	DIMENSIONS.	COST.
U. S. Government	800 ft. by 200 ft.	\$400,000
Education	400 ft. by 600 ft.	350,000
Art	836 ft. by 450 ft.	1,000,000
Liberal Arts	525 ft. by 750 ft.	475,000
Manufactures	525 ft. by 1,200 ft.	719,399
Varied Industries	525 ft. by 1,200 ft.	604,000
Textiles	525 ft. by 700 ft.	319,399
Machinery	525 ft. by 1,000 ft.	496,957
Electricity	525 ft. by 750 ft.	389,940
Transportation	525 ft. by 1,300 ft.	700,000
Agriculture	600 ft. by 1,600 ft.	800,000
Horticulture	400 ft. by 800 ft.	200,000
Forestry and Fisheries	400 ft. by 600 ft.	350,000

4. Find the area in square feet covered by each building.

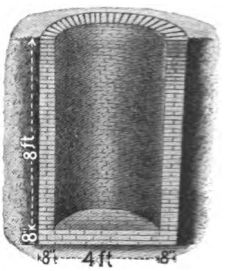
5. Find the total number of acres covered by all the buildings combined.

6. Find the total cost of these buildings. Find the cost of each building per square foot of surface covered.

1. An 8-sided tent has the dimensions indicated; how many square yards of canvas are required to make it?



2. How many cubic feet of masonry are there in the wall and bottom of the manhole half of which is shown in the figure?

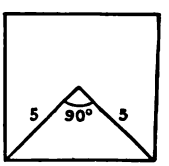


3. How many cubic feet of water would fill the manhole mentioned in Exercise **2**.

4. One of the equal angles of an isosceles triangle is 75°; find the angle at the vertex.

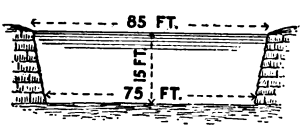
5. What is the length of the side of a square whose area is 361 sq. in.?

6. Find the area of this square, the dimensions being given in inches.



7. A house covers a rectangular piece of ground 60 ft. by 40 ft. The ridge pole runs lengthwise and is 15 ft. above the level of the top of the walls. The roof extends 1 ft. beyond the walls in all directions, measured on the roof. Find the area of the roof.

8. A canal is to be constructed, 5 mi. in length and whose cross-section has the dimensions indicated in the figure.



A contractor, wishing to bid, estimates that it will cost him $12\frac{1}{2}¢$ a cubic yard for excavation and $30¢$ a load for hauling the earth away. The earth is hauled away in carts 4 ft. wide, $2\frac{1}{2}$ ft. deep and 8 ft. long. It is estimated that the loose earth occupies 10% more space than the solid earth. The contractor wishes to allow 25% for incidental expenses and profit; how much should he bid?

1. If a cylinder weighing 48 lb. were tapered at each end $\frac{1}{8}$ of the total length, find the weight of the remaining solid.

2. A cone and a cylinder have equal bases and volumes; what is the ratio between their altitudes?

3. If the perimeter of the base of a prism is 16 and its altitude is 5, what is its lateral surface?

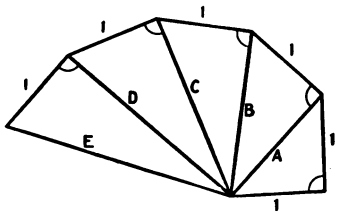
4. If the lateral surface is 120 and the altitude 6, what is its perimeter?

Fill out the blanks for prisms:

	5.	6.	7.	8.	9.	10.	11.
Lateral surface:	4	—	180	—	$\frac{15}{4}$	$2\frac{1}{2}$	$12m^2$
Altitude:	7	9	—	$\frac{1}{2}$	$\frac{3}{4}$	—	—
Perimeter of base:	—	20	15	$\frac{1}{8}$	—	$1\frac{2}{3}$	$3m$

	12.	13.	14.	15.	16.	17.	18.
Volume:	—	—	—	200	640	800	130
Altitude:	12	20	30	20	16	—	—
Area of base:	16	25	14	—	—	25	26

19. The triangles in this figure are right triangles. Find the lengths of A, B, C, D, E.



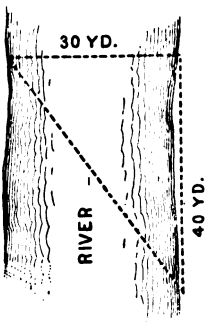
20. If the perimeter of the base of a right pyramid is 20 in. and the slant height is 18 in., what is the lateral surface?

21. What is the perimeter of the base, if the lateral surface is 200 sq. ft. and the slant height 20 ft.?

Fill out the blanks for pyramids:

	22.	23.	24.	25.	26.	27.	28.
Slant height:	28	—	16	75	—	3.5	—
Lateral surface:	—	400	320	375	625	—	$25d^2$
Perimeter of base:	30	40	—	—	15	8.2	$5d$

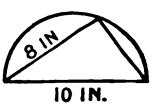
1. A swimmer in crossing a stream 30 yd. wide was carried down 40 yd. by the current; how many yards did he actually go?



2. Find the length of one of the equal sides of an isosceles triangle whose base is 8 in. and altitude 3 in.

3. The length of one of the equal sides of an isosceles triangle is 13 ft. and the base is 10 ft. Find the altitude of the triangle. Find its area.

4. The diameter of a circle and one chord have the lengths and positions shown in the figure. The third chord completes a right triangle; find the length of the third chord.



5. Find the volume of a solid in the form of a cube surmounted by a square pyramid just covering one face of the cube; the whole object is 15 ft. high and the edge of the cube is 10 ft.

Determine the numbers to fill the blanks:

	VOLUME OF PYRAMID.	AREA OF BASE.	ALTITUDE.
6.	432 cu. ft.	———	12 ft.
7.	96 cu. in.	36 sq. in.	———
8.	———	225 sq. ft.	15 ft.

9. What is the volume of a cone whose base contains 18 sq. ft. and whose altitude is 27 ft.?

10. How may the lateral area of a cone be found? What is the lateral area of a cone of radius 6 in. and slant height 20 in.?

11. The basin of a fountain was in the form of a frustum of a cone. The radius of the top was 6 ft., of the bottom 4 ft., and the depth was 4 ft.; find the volume of the basin in cubic feet.

12. How many gallons did the basin of Exercise 11 hold ($7\frac{1}{2}$ gal. $= 1$ cu. ft.)?

XXV

DENOMINATE NUMBERS

TABLES

Linear

12 inches (in.) = 1 foot (ft.)
3 feet (ft.) = 1 yard (yd.)
16.5 feet = 1 rod (rd.)
320 rods (rd.) = 1 mile (mi.)
1760 yards (yd.) = 1 mile
5280 feet = 1 mile
6 feet = 1 fathom

Time

60 seconds (sec.) = 1 minute
60 minutes (min.) = 1 hour (hr.)
24 hours (hr.) = 1 day (da.)
7 days (da.) = 1 week (wk.)
365 days or
12 months (mo.) = 1 year (yr.)
10 years = 1 decade
10 decades = 1 century

Liquid

2 pints (pt.) = 1 quart (qt.)
4 quarts (qt.) = 1 gallon (gal.)
231 cu. in. = 1 gal.

Counting

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (gt. gro.)

Dry

2 pints (pt.) = 1 quart (qt.)
8 quarts (qt.) = 1 peck (pk.)
4 pecks (pk.) = 1 bushel (bu.)
32 quarts = 1 bu.
2150.4 cu. in. = 1 bu.

Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.)
100 pounds (lb.) = 1 hundredweight
2000 pounds = 1 ton (t.)
2240 pounds = 1 gross ton
7000 grains = 1 pound

Value

10 cents (¢) = 1 dime
10 dimes = 1 dollar
100 cents = 1 dollar ($)
10 dollars = 1 eagle
20 dollars = 1 double eagle

Angle

60 seconds (″) = 1 minute (′)
60′ = 1 degree (°)
90° = 1 right angle (rt. ∠)
180° = 1 straight angle (st. ∠)
360° = 1 circumference

Square

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet (sq. ft.) = 1 square yard (sq. yd.)
30¼ sq. yd. = 1 square rod (sq. rd.)
160 sq. rd. = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
1 sq. mi. = a section
36 sq. mi. = a township
100 sq. ft. = a square (of roof, etc.)

Cubic

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet (cu. ft.)	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord
1 cubic yard	= 1 load (of earth, etc.)
24¾ cubic feet	= 1 perch (stone)

Surveyors' Measures of Length

7.92 inches	= 1 link (li.)
100 links	= 1 chain (ch.)
80 chains	= 1 mile

The public lands of the United States are surveyed by use of the chain.

Surveyors and engineers now generally use a steel tape graduated in feet and inches or in feet and tenths of a foot.

Counting Sheets of Paper

24 sheets	= 1 quire
20 quires, or 480 sheets	= 1 ream

Apothecaries' Weight

20 grains (gr.)	= 1 scruple (sc. or ℈)
3 scruples	= 1 dram (dr. or ʒ)
8 drams	= 1 ounce (oz. or ℥)
12 ounces	= 1 pound (lb.)
5760 grains	= 1 pound

The table of apothecaries' weight is used in selling drugs at retail. The metric system is taking its place with pharmacists.

Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt. or dwt.)
20 pennyweights	= 1 Troy ounce
12 Troy ounces	= 1 Troy pound
480 grains	= 1 Troy oz.
5760 grains	= 1 Troy lb.
437.5 grains	= 1 avoirdupois oz.
7000 grains	= 1 avoirdupois lb.

Troy weight is used in weighing precious metals.

For the metric system and measures of foreign countries see the Supplement, pp. 231–245.

SUPPLEMENT

METRIC SYSTEM

The **International,** or **Metric,** System of measures has repeatedly been mentioned in what precedes. It is called international because it has been adopted by the following 43 governments: *

Argentina	France	Nicaragua
Austria-Hungary and territories	French Colonies (including Madagascar)	Norway and Sweden
Belgium	Germany	Ottoman Empire
Bolivia	Greece	Peru
Brazil	Guatemala	Philippines
Bulgaria	Haiti	Porto Rico
Central America	Holland and dependencies	Portugal, Azores, and Madeira
China (28 ports)	Honduras	Roumania
Chili	Italy and dependencies	Russia
Colombia	Japan	Salvador
Costa Rica	Java	San Domingo
Cuba	Mauritius and dependencies	Servia
Ecuador	Mexico	Spain and colonies
Egypt		Switzerland
Finland		Uruguay

The system has been legalized in the United States and Great Britain.

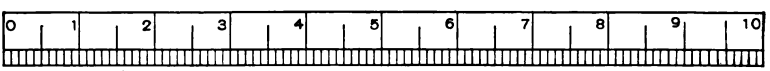
It has not been legalized in Morocco, Denmark, Persia, Siam, the interior of China and among the African races.

Although the international system of measures has not yet been adopted by the United States, our large and growing international commerce requires acquaintance with it. The system is already used to a large extent in the United States by manufacturers for export and by scientists.

* The statements of this page and a few of the exercises following are taken from the report of the Committee on Coinage, Weights and Measures of the 57th Congress (1902).

Linear Measure. In the international system a unit of each denomination is always 10 times as large as the unit of the next lower denomination. This makes it possible to use decimals constantly, and hence the system is often called the *decimal* system.

LINEAR MEASURE.	IN COMMON USE.
10 millimeters = 1 centimeter	10 millimeters = 1 centimeter
10 centimeters = 1 decimeter	100 centimeters = 1 meter
10 decimeters = 1 meter	1000 millimeters = 1 meter
10 meters = 1 dekameter	1000 meters = 1 kilometer
10 dekameters = 1 hektometer	
10 hektometers = 1 kilometer	



One Decimeter.

Abbreviations. The following are standard abbreviations:

mm. for millimeter	m. for meter
cm. " centimeter	km. " kilometer
dm. " decimeter	

Reading. For small lengths the millimeter is used. A board would be described as 23 millimeters thick, not 2 centimeters 3 millimeters, just as we say a 10-qt. bucket, not a 2-gal. 2-qt. bucket. For greater lengths, centimeters, meters, and kilometers are the usual units. Thus:

36 cm.	is read	36 centimeters,
7.32 m.	" "	7 and 32 hundredths meters,
465 mm.	" "	465 millimeters,
273.5 m.	" "	273 and 5 tenths meters,
56.47 km.	" "	56 and 47 hundredths kilometers.

Only one unit is commonly used to express a magnitude.

Read the following:

1. 3.5 m.; 84 mm.; 27 cm.
2. A board is 18 mm. thick.
3. A picture is 27 cm. by 32 cm.
4. A remnant of silk contains 3.27 m.
5. From Paris to Calais is 295.32 km.

1. What is the cost of 7 m. of cloth at $1.80 a m.?

2. What is the cost of 9.2 m. of ribbon at $.35 a m.?

3. What is the cost of 17.46 m. of lace at $.55 a m.?

4. How many square meters of carpet will it take to cover a floor 6 m. long and 5 m. wide? One 7.2 m. long and 3.45 m. wide?

5. A rope is 5.36 m. long; how many cm. long is it?

6. How many cm. are there in 16 m.? In 60 mm.?

7. Which divisions of the scale on the opposite page are centimeters? Which are millimeters?

8. How many rounds are there in a ladder 12 m. long when the rounds are 40 cm. apart?

9. How many meters of ribbon would be required to make 35 badges 20 cm. in length?

10. If the current of a river flows 2,160 m. an hour, how many meters does it flow in a minute?

11. 1 meter = — decimeters. 1 m. = — cm.

12. 1 cm. = — mm. 1 dm. = — mm. 1 mm. = .001 of a —. 1 cm. = .01 of a —. 1 dm. = .1 of a —.

13. What are the common international units of length?

14. The meter is 39.37 in. long. Express a decimeter in inches; a millimeter.

15. The United States postal service accepts packages not over 30 cm. in length, 20 cm. in width and 10 cm. in thickness. Express the dimensions in inches.

16. Express the following barometric readings in centimeters: 29.8 in.; 30.1 in.; 28.6 in.

17. Taking 1 m. = $3\frac{1}{4}$ ft., how many meters in a mile (to nearest meter)?

18. The distance from Paris to Rouen is 136 km.; express this distance in miles (to nearest mile).

Square Measure. The common units are the **square meter** (sq. m.), **square centimeter** (sq. cm.), the **square millimeter** (sq. mm.), and the **square dekameter** (are), the **square hektometer** (hektare, ha.), and the **square kilometer** (sq. km.).

TABLE OF SQUARE MEASURE.

100 square millimeters = 1 square centimeter
100 square centimeters = 1 square decimeter
100 square decimeters = 1 square meter
100 square meters = 1 square dekameter
100 square dekameters = 1 square hektometer
100 square hektometers = 1 square kilometer

1. How many sq. mm. are there in 2 sq. cm.? How many sq. cm. are there in 4 sq. m.?

2. How many sq. m. are there in 1 are? How many ares are there in 2 hektares?

3. How many hektares are there in a sq. km.?

4. If a sq. cm. of gold leaf is worth 3¢, what is the cost of 10 sq. m.?

5. A hektare is about 2.47 acres; 30 ha. are about how many acres? 50 hektares are about how many acres?

6. At $2.50 an are, what is the cost of 25 ha. of land?

7. How many sq. m. are there in the surface of a street 20 m. wide and 5 km. long?

8. A floor is 5.2 m. by 8.4 m.; how many tiles 7 cm. by 13 cm. would be required to pave it?

9. A lot contains 2.7 ares. The owner refused an offer of $1,800 an are for it. Afterward he sold it for $20 a sq. m. Did he gain or lose, and how much?

10. The distance from Paris to Orleans is 121 km. The railway is a double track and the rails are 4 m. long; how many rails are used?

11. The roadbed of the railway just mentioned is 15 m. wide; how many hektares does it contain?

Cubic Measure. The cubes of the various linear units are used as measures of volume. The most important are the **cubic meter** (cu. m.), the **cubic decimeter** (cu. dm.), the **cubic centimeter** (cu. cm.), and the **cubic millimeter** (cu. mm.).

TABLE OF CUBIC MEASURE.

1000 cubic millimeters	= 1 cubic centimeter
1000 cubic centimeters	= 1 cubic decimeter
1000 cubic decimeters	= 1 cubic meter
1000 cubic meters	= 1 cubic dekameter
1000 cubic dekameters	= 1 cubic hektometer
1000 cubic hektometers	= 1 cubic kilometer

The cubic meter is called a stere when used in measuring wood.

1. If a cu. m. of granite weighs 7 metric tons, what is the weight of a block 5.5 m. long, 3.75 m. wide and 2.4 m. thick?

2. How many cu. m. of earth are there in an excavation 4 m. long, 6.5 m. wide and 2.2 m. deep?

3. How many wagon loads of earth of 1 cu. m. each are removed in excavating a cellar 5 m. long, 4.5 m. wide and 2.8 m. deep?

4. How many cu. cm. are there in a package 30 cm. by 20 cm. by 10 cm.?

5. How many steres of wood are there in a pile 10.5 m. long, 1 m. wide and 2.5 m. high?

6. At $7.50 a cu. m. what is the cost of a bridge pier 10.5 m. wide, 3 m. thick and 13 m. high?

7. How many cubic meters are there in the capacity of a car 10 m. long, 2.5 m. high and 2 m. wide?

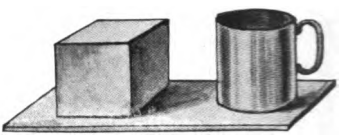
8. On ocean steamers $\frac{1}{2}$ cu. m. is sometimes allowed for a passenger's baggage. A traveler has a box 42 cm. by 48 cm. by 76 cm.; a trunk 32 cm. by 80 cm. by 50 cm., a chest 45 cm. by 52 cm. by 62 cm., and a box 30 cm. by 40 cm. by 105 cm.; has he excess baggage? If so, how much? What is the charge for the excess at $10 a cu. m.?

Measure of Capacity. The **liter** is the unit for liquid and dry measures. Its capacity is 1 cubic decimeter.

In countries where the metric system is in use, the liter corresponds to the English quart measure. That is, just as milk is sold in New York City by the quart, it is sold in Paris by the liter.

THE TABLE OF CAPACITY.

10 centiliters = 1 deciliter
10 deciliters = 1 liter (l.)
10 liters = 1 dekaliter
10 dekaliters = 1 hektoliter (hl.)
10 hektoliters = 1 kiloliter (kl.)



CU. DM. LITER

1. How many liters of water are there in a tank 3 dm. by 18 dm. by 7.5 dm.?

2. A water tank has a capacity of 1 cu. m.; how many liters does it hold?

3. How many hl. of grain are there in a bin 4 m. by 5 m. by 2 m.?

4. A grocer bought beans at $2.00 a hl. and sold them at 3¢ a l.; what per cent did he gain?

5. A milkman sold 5 kl. of milk in a month at 5¢ a l.; to how much did the sales amount?

6. A tank 1 m. wide, 5.5 m. long, and 20 cm. deep is full of water; how many liters does it hold?

7. If the rainfall is 11 cm. for a certain month, how many cu. cm. of water fall on an are of land? How many kiloliters?

8. 2.5 kl. of olive oil was put up in bottles holding .5 l. each; how many bottles were used?

9. A cylindrical tank of radius .52 m. is filled with water to a depth of 2.58 m.; how many liters of water are there in the tank?

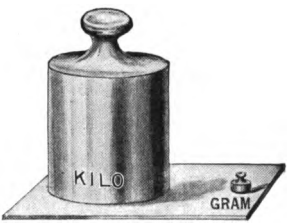
10. 2 deciliters are what part of a liter? 7 centiliters?

Measure of Weight. The weight of a liter of water at a fixed temperature is taken as a unit of weight and called a **kilogram,** or simply **kilo.**

The kilo is about 2.2 lb. and is used in the same manner that the pound is used in the United States.

THE TABLE OF WEIGHT.

10 centigrams = 1 decigram
10 decigrams = 1 gram (g.)
10 grams = 1 dekagram
10 dekagrams = 1 hektogram
10 hektograms = 1 kilogram
1000 kilograms = a metric ton = the weight of 1 cu. m. of water.



The weights most commonly used are the kilo and gram.

1. .5 of a kilogram is how many grams?

2. A cubic decimeter of water weighs how many grams? How many kilograms?

3. Alcohol is .83 as heavy as an equal volume of water. A liter of alcohol weighs how many grams? What part of a kilogram?

4. A tank holding 5 cu. m. of water weighs how many metric tons?

5. A tank whose volume is 1 cu. m. contains how many liters? How many kilos does the water weigh?

6. If the weight of ice is .92 of that of water per volume, what is the weight in kilos of a block of ice .2 m. by .2 m. by 5 m.?

7. Petroleum is .7 as heavy as water per volume; what is the weight in pounds of 1 cu. m. of petroleum?

8. Express 5 centigrams as grams; 3 decigrams as grams and add the results.

9. Express as kilos, 6 hektograms + 4 dekagrams + 3 grams.

The International system and the Anglo-American system contrasted:

1. *Add:*

ANGLO-AMERICAN.		
YD.	FT.	IN.
12	2	7
9	1	8
14	0	11
6	2	5
7	1	10
51	0	5

Add:

INTERNATIONAL.
METERS.
12.27
9.18
14.11
6.25
7.20
49.01

2. *Reduce to inches:*

3 mi.
167 rods
3 yd.
2 ft.
9 in.

Ans., 223,287 in.

Reduce to millimeters:

5 kilometers
6 hektometers
7 dekameters
1 meter
5 decimeters
1 centimeter
2 millimeters

Ans., 5,671,512 mm. (by inspection).

3. Find the contents of a tank:

8 ft. 4 in. long
5 ft. wide
2 ft. 5 in. deep

Ans., 100 cu. ft. 1,200 cu. in.

2.5 m. long
1.52 m. wide
80 cm. deep

Ans., 3.04 cu. m.

4. Find how much the tank holds:

In gallons: (1 gal. = 231 cu. in.).

Ans., $753\frac{55}{231}$ gal.

In liters:

Ans., 3,040 liters.

5. Find the weight of water filling the above tank.

Ans., 3 tons 264 lb.

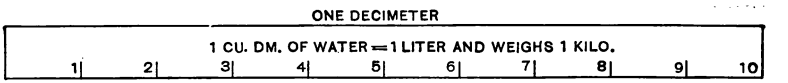
Ans., 3.04 metric tons (by inspection).

6. What is the pressure on the bottom when the tank is filled with water?

Ans., 1.044 lb. per sq. in.

Ans., 80 grams per sq. cm. (No computation needed, since 1 cu. cm. of water weighs 1 g. and the tank is 80 cm. deep.)

The relation between the units of measure:



1. If the metric system were used altogether in this country, what points of superiority would it have over the Anglo-American system?

2. A meter being about $1\frac{1}{12}$ yd., how many square yards are there in a square meter?

3. What is the cost of making a cement sidewalk 0.875 km. long, 3 m. wide at 65¢ a square meter?

4. 1 cu. dm. of water weighs about 2.2 lb.; what is the weight of the water in a tank containing 5 cu. m.?

5. 4 steres make a wagon load of wood; how many loads of wood are there in a pile $10\frac{1}{2}$ m. long, 1 m. wide, and $2\frac{3}{4}$ m. high?

6. The Eiffel tower is 300 m. high; about how many feet high is it?

7. Since a dm. of water weighs 2.2 lb. and granite is 2.7 times as heavy as water, what is the weight of a granite block 2 m. by 3 m. by $5\frac{1}{2}$ m.?

8. Copper is 8.8 times as heavy as water per volume; a cube of copper of edge 10 cm. weighs how many kg.?

9. How many square meters are there in the floor of a room 6 m. long and 5 m. wide? How many tiles 1 dm. square will it take to cover it?

10. The specific gravity of hydrogen referred to air is 0.07 and the weight of 1 liter of air is 1.293 g.; what is the weight of 1 liter of hydrogen?

11. A cylindrical granite shaft is 2 m. in diameter and 10 m. high. If the specific gravity of the stone is 7.8, what is the weight of the shaft?

TABLE OF VALUES OF FOREIGN MONEYS.

Country.	Monetary Unit.	Value in United States Gold, as fixed by the U. S. Treasury.
Canada	Dollar	$1.00
Great Britain	Pound sterling	4.8665
France	Franc	.193
Belgium	Franc	.193
Switzerland	Franc	.193
Italy	Lira	.193
Spain	Peseta	.193
Greece	Drachma	.193
Germany	Mark	.238
Austria-Hungary	Crown	.203
Cuba	Peso	.926
Mexico	Dollar (silver)	.443

The prices at which foreign money is actually bought and sold will vary slightly from the above official values. The market price is called the *rate of exchange.*

CURRENCY OF GREAT BRITAIN.

1 pound sterling	= 20 shillings	£1 = 20 *s.*
1 shilling	= 12 pence	1*s.* = 12 *d.*
1 penny	= 4 farthings	1*d.* = 4 far.

The rate of exchange for sterling is usually expressed by stating in United States money the amount for which £1 sterling can be bought or sold.

Thus, "sterling exchange 4.88" means that $4.88 are paid for £1.

Fractions of the smallest current coin, as the cent of United States money, are to be rejected if less than ½, otherwise to be counted as a whole.

1. When sterling exchange is 4.88, what will a draft on London for £200 cost?

2. If sterling exchange is quoted at 4.87½, how much English money can be procured for $1,000?

3. A merchant owes a London firm £1,421 14*s.* 6*d.* (usually written, £1,421–14–6); what must he pay for a draft for this amount at 4.87½?

1. The English parliament has voted the King an annual income of £470,000. Make an approximate estimate of the equivalent in United States money, taking £1 = $5.

2. The income of the government of the United Kingdom for a recent year is given in the table; fill the blank:

Taxes	£98,496,000
Post-offices	13,192,000
Telegraphs	3,390,000
Remainder	
Total	£128,633,000

3. Make an approximate estimate of the equivalent of each item of the above table in United States money.

4. Determine the equivalent to nearest cent in United States money of each item, using £1 = $4.8665.

5. The savings banks deposits average as follows per capita of population in the countries named:

	£	s.	d.		£	s.	d.
Denmark	15	11	6½	United States	6	4	10½
Switzerland	13	0	2¾	Austria-Hungary	5	8	3¾
Germany	7	10	6¾	France	4	8	8½
Norway	7	8	7½	Great Britain	4	2	5¾
Australia	7	6	4¾	Canada	2	5	2¾

Express each amount in United States money.

6. Express in United States money the items of the following table; fill the blanks:

GREAT BRITAIN.	IMPORTS FROM.	EXPORTS TO.
British Possessions	£99,433,955	£83,426,761
United States	126,062,155	14,716,489
France	51,396,793	13,706,246
Germany	28,534,159	22,525,987
Others		
Total	£470,378,583	£233,359,987

The unit is the **franc** (fr.); it is divided into 100 **centimes** (pronounced *sonteems*). 100 centimes equal 1 **franc.**

Hence, hundredths of a franc are read centimes.

Thus, 14.25 fr. = 14 francs, 25 centimes; 0.60 fr. = 60 centimes.

It is customary in France and Germany to write a zero before the decimal point when the number is a pure decimal.

Equivalents. The price of French money is usually quoted by stating how many francs are worth one dollar.

Thus, if francs are quoted at 5.15¼, 5 francs and 15¼ centimes are equivalent to $1. For approximate estimates, 1 franc = 20¢, 5 francs = $1.

1. At 5.15 how many francs can be procured for $500?

2. How much will a draft for 2,000 francs cost at 5.15⅜?

Find in francs the total cost of the following purchases:

3. 23.64 meters of silk at 7.25 francs per meter.

4. 8.5 meters of velvet at 9.95 francs per meter.

5. 127.85 meters of ribbon at 0.60 franc per meter.

6. The official estimates of governmental revenue in France were as follows for a recent year; fill the blank:

Direct taxes	522,596,614 fr.
Indirect taxes	2,117,391,250 fr.
Government monopolies (tobacco, matches, gunpowder, posts, telegraphs, telephones, etc.)	728,651,830 fr.
Various	
Total	3,551,570,497 fr.

7. Imports and exports of France in a recent year:

COUNTRY.	IMPORTS.	EXPORTS.
	Million francs.	*Million francs.*
British Isles	669	1,239
United States	460	254
Germany	412	460
Others		
Total	4,409	4,079

What per cent of the total was the item for the United States in each case? Supply the missing numbers.

The unit is the Reichsmark or **Mark** (M.). It is divided into 100 Pfennigs (pfgs.). 1 mark = 100 pfgs.

Hence, hundredths of a mark are read pfennigs.

Thus, 4.01 M. = 4 marks and 1 pfennig. 0.70 M. = 70 pfennigs.

Equivalents. The rate of exchange for German money is usually stated by naming in cents the equivalent of 4 marks.

Thus, when marks are quoted at $95\frac{7}{8}$, 4 marks are sold for $95\frac{7}{8}$ cents.

For approximate estimates, take one mark as 25¢.

1. When marks are quoted at $95\frac{1}{2}$, what will be the cost of a draft on Berlin for 20,000 M.?

2. If marks cost $95\frac{3}{4}$, how many marks can be procured for $1,200?

Find the total cost in marks of the following purchases:

3. 8.6 meters of calico at 0.45 M. per meter.

4. 14.5 meters of muslin at 0.65 M. per meter.

5. 12 meters of cloth at 5.28 M. per meter.

6. 0.6 kilos of butter at 3.30 M. per kilo.

7. Make approximate estimates of the equivalents in United States money of the prices and of the results of Exercises 3 to 6.

8. The expenditures of the German Empire in a recent year were; find the missing number:

	MARKS.
Army	665,850,576
Navy	172,207,975
Postal and telegraph	355,910,050
Railways	79,925,610
Administration	586,358,408
Other purposes	
Total	2,069,825,412

9. Estimate the equivalent amounts of United States money for each item of Exercise 8.

Travelers' Money-Orders. Travelers in foreign countries often carry *bank* or *express money-orders* instead of cash. The following is a bank money-order:

This order cost the purchaser, Mr. Pierce, fifty dollars. The amount of money that he will receive if he presents it at any of the banks in England named on the back of the order is shown on its face to be £10 4*s*. 1*d*.

1. When 1 pound of English money is worth $4.866, what is £10 4*s*. 1*d*. worth?

Thus, the traveler does not generally receive the equivalent of $50 when he cashes the order.

2. The cash value of the above order in Germany is 206.25 marks. At 23.8¢ a mark, find the equivalent of this amount in United States money.

3. The cash value of the same order in France is 256.25 francs. At 19.3¢ a franc, find the equivalent of this amount in United States money.

4. Find in English money the cash value of 10 fifty-dollar express money-orders at the rate given in the above order; find similarly their cash value in Paris; in Berlin.

5. Find the equivalent in United States money of the order shown above, if cashed in Austria, the crown being 20.3¢.

To find the approximate equivalents of amounts of foreign money in United States money use the following:

One pound = $5.00.	One mark (M.) = $.25.
One shilling = $.25.	One franc (fr.) = $.20.

1. An American in London sees the following prices in the shop windows: hats, 5/9 (5*s.* 9*d.*); gloves, 2/11½; suit, 65/0; overcoat, 2–10–6 (£2 10*s.* 6*d.*); necktie, 1/5½. What would be the approximate equivalents in United States money?

2. The following purchases were made in London:

2½ yd. lace at 18/6 a yard.	16 yd. silk at 10/3 a yard.
7⅞ yd. velvet at 13/9 a yard.	50 yd. ribbon at 1/3 a yard.

Find the total cost in English money and find the approximate cost of each item in United States money.

3. Find the exact cost of the whole purchase mentioned in Exercise 2, exchange being quoted at $4.88⅝.

4. A man offered a pound (often called a sovereign) in payment of a book and received 12*s.* 6*d.* in change; what did the book cost?

5. A person imported some books from Berlin; the bill was 200 M.; about how many dollars was this?

6. A traveler paid 15.50 fr. for a railway ticket; about how many dollars and cents did the ticket cost?

7. A man's hotel bill in Paris was 67 fr.; about how many dollars was this?

8. A man spent 1,300 M. while sightseeing in Germany; about how many dollars was this?

9. A traveler bought an Oriental rug for 240 fr. Find the approximate equivalent of the price in United States money.

10. It cost 100,000 fr. to lay out one of the national parks of France; how many dollars was this?